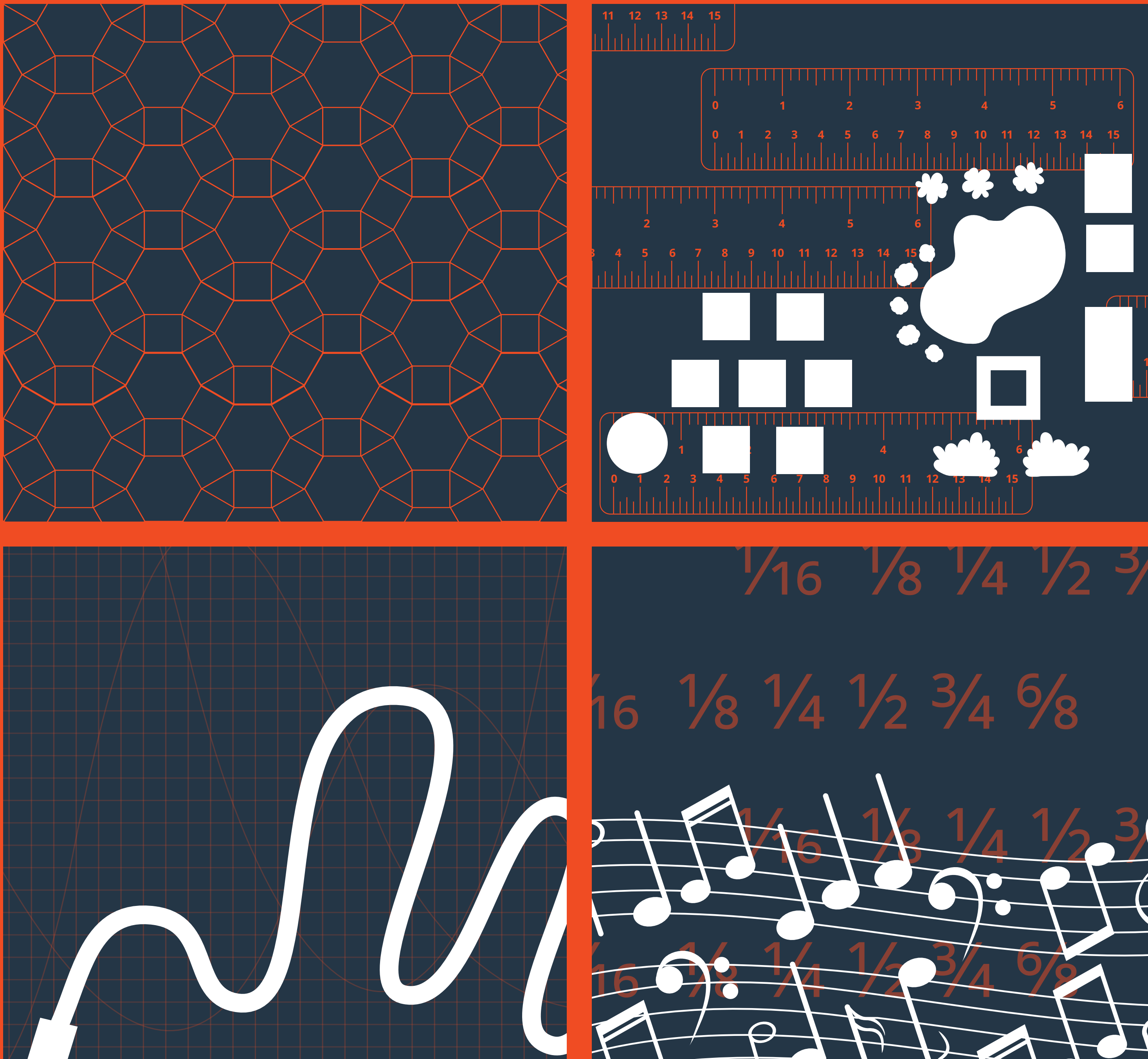

Afterschool Math Plus

fhi360

Afterschool Math Plus

Revised Edition

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Afterschool Math Plus

Theme +1: ArtMath

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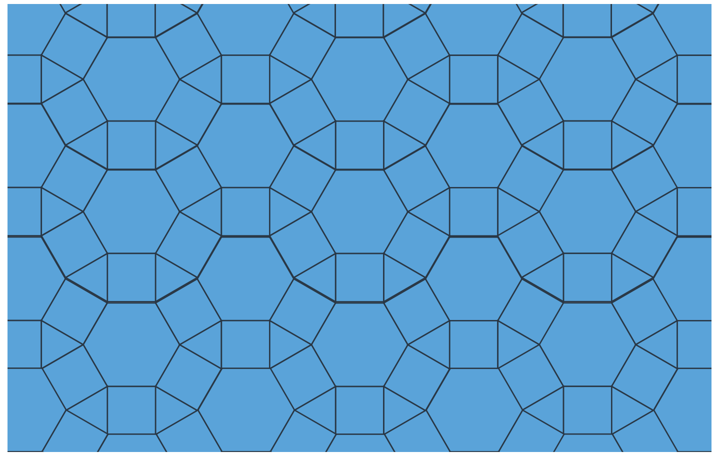
Introduction



Why is this theme called ArtMath? It's because art and math are so connected to each other. Artists use math all the time. They measure the space on their canvas and decide the scale for their work. Some artists incorporate principles of graphic design in their work, which requires knowledge about geometric shapes, tessellations, and repeating patterns.

Artists often use the same tools that are used in math—rulers, yard sticks, measuring tapes, grid paper, even protractors.

Mathematicians are creative, too. Like artists, they also create designs on paper and draw things to scale and use a variety of measurement tools in their work. They create computer models using many different colors to describe their work.

In different ways, artists and mathematicians bring order and beauty into our lives. Both may use abstractions to help us think about things in new ways. A mathematician may be inspired by an artist or vice versa.

In the ArtMath theme, students explore the many ways in which art and math interconnect. They create kaleidoscopes and look at M.C. Escher's art to understand tessellations—a concept found in both math and art. They experience the role of scale, geometry, tessellations (a special type of repeated patterns), symmetry, measurement—all essential math skills and concepts. They also learn about asymmetry through the art of Piet Mondrian.

As a culminating activity, students create an exhibit that demonstrates the kind of math they have used to create their art.

Equity

Artists come from all walks of life and from across every culture. Art has no age limit, and, while not all artists are famous, you may be looking at the next Escher or Mondrian right in your very class! Encourage all students to be creative and enjoy their works of art.

NCTM Math Standards

Content Standards

- **Geometry:** Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes; understand relationships among the angles and the length of the sides of similar objects; explore congruence and similarity; and make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.
- **Algebra:** Understand patterns, relations, and functions; describe, extend, and make generalizations about geometric and numeric patterns; and represent and analyze patterns and functions, using words, tables, and graphs.

Process Standards

- **Problem Solving:** Apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematical problem solving.
- **Communication:** Communicate mathematical thinking coherently and clearly to peers, teachers, and others; and use the language of mathematics to express mathematical ideas precisely.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

Inclusion

A visual arts theme obviously presents special challenges for educators working with students who are visually impaired. Every effort should be made to make materials large and bright for students with low vision, and tactile and/or auditory for students who are blind. There are many ways to make materials tactile, including raised glue lines, wax-covered yarn, and sandpaper, among many others. These and other methods are explained more fully in each activity. Most importantly, be sure to fully describe all items displayed and discussed, and be sure to ask students what other strategies they would find helpful. While these techniques are most important for students with disabilities, they are helpful for all students, meeting the needs of many different learning styles.

Cultural Links

Beautiful art can be found in all cultures. When looking for samples of tessellations, symmetry, and asymmetry, consider using art that represents the cultures of both the students you are working with and those not represented in the group.

Literacy and Math Identity

Students will learn about people from diverse backgrounds who have made careers in art and how math figures in their work. And they will learn about new and creative career options that will be open to them if they continue to take math courses beyond the basics.

The role models for this theme all have a connection to math and art. M.C. Escher used tessellations to create much of his art. Piet Mondrian instinctively used lines, blocks of color, and the golden ratio or golden rectangle (see the Glossary) in his work. One of Mondrian's best-known paintings is Broadway Boogie Woogie, which looks like a New York City street grid. Alma Woodsey Thomas is an abstract artist who uses her math skills to create art. She became the first African American artist to have a one-woman show at the Whitney Museum of American Art in New York City. David Brewster, a 19th century English philosopher and scientist, used mathematics to invent the kaleidoscope to demonstrate optical principles. Doris Schattschneider creates art using tiling—a form of tessellations. And Gordon Sasaki, an artist and educator, uses math to create art and teaches about the connections between art, math, and identity.

At the Museum

During their sessions at the museum, students measure, sketch, and design their very own art work in the style of Escher or Mondrian, two well-known 20th century artists. Museum staff and afterschool staff collaborate to plan a culminating event that highlights the students' work—a museum "opening" or a public display in a community space.

Family Connections

When you start this theme, send out the included family letter. It will give families the tools to support student learning and create a higher level of family involvement.

During the culminating event, students shared the artwork projects they had created at their afterschool center and at the museum. Families were proud to hear their children talk about tessellations, rotations, asymmetry, and other math concepts they had learned through making drawings in the style of Escher and Mondrian. The math of tessellation and rotation was further demonstrated as family members looked through the changing patterns in the kaleidoscopes the students had created as the first activity in the ArtMath theme.

Remember to:

- ☐ **Check out the resources.**
- ☐ **Review the glossary.**
- ☐ **Send out the family letter when you start the theme.**

Dear Families,

Mathematics and art are connected in so many ways—but we don't often think about them. Artists use math all the time. They measure the space on their canvas and decide the scale for their work. Often, they use geometric shapes, patterns or repeating designs in their work.

Mathematicians are creative, too. They create designs on paper and use a variety of measuring tools in their work. They create computer models using many different colors and techniques to describe their work.

This After-School Math PLUS theme is called ArtMath! There are five fun activities that include looking at the work of two modern artists, M.C. Escher and Piet Mondrian. Escher used math concepts as the basis for his art, and Mondrian's art involves color blocks, rectangles, and asymmetry. As part of this theme, your child will create art in the style of these two artists, take a field trip (to a museum, library, or, virtually, on the computer) to see their work, and use geometry and algebra to build a kaleidoscope.

In addition, a series of activities will help your child think about careers and the math skills needed for success. The activities will also broaden your child's view of who is doing math and introduce some interesting math role models.

There are several ways you can be a part of this experience to enhance the understanding for your child:

- Visit an art museum with your child. Look for the math in the art—did the artist use geometric shapes or patterns?

- Go online with your child to look for samples of Escher's and Mondrian's art.

- Let your child teach you about the math in art! Ask them to show you the asymmetric patterns in Mondrian, the symmetry in Escher, or how to build a kaleidoscope from a recycled potato chip can.

After several sessions of hands-on, minds-on experiences, the afterschool group will be going to a local science museum to build an exhibit. Please come to the opening to see your child's ArtMath.

Sincerely,

Queridas Familias,

Las matemáticas y el arte están conectados en muchas maneras—pero no lo pensamos a menudo. Artistas usan las matemáticas todo el tiempo. Miden el espacio en su lona o canvas y deciden la escala para su trabajo. A menudo, usan las formas geométricas, patrones o diseños que repiten en sus pinturas.

Los matemáticos son creativos, también. Crean diseños en papel y utilizan una variedad de herramientas de medida en su trabajo. Crean modelos en la computadora usando diversos colores y técnicas para describir su trabajo.

¡Este tema del programa Matemáticas Despues-de-Escuela y MÁS se llama ArtMath (Arte-Matemáticas)! Hay cinco actividades divertidas que incluyen mirando el trabajo de dos artistas modernos, M.C. Escher y Piet Mondrian. Escher utiliza los conceptos de las matemáticas como la base de su arte, y el de Mondrian implica bloques y rectángulos en color y asimetría. Como parte de este tema, su niño/niña creará arte en el estilo de estos dos artistas, tomará un viaje (a un museo, una biblioteca, o, virtualmente, en la computa- dora) para ver su trabajo, y usará geometría y álgebra para construir un kaleidoscopio.

Además, una serie de actividades ayudará a su niño/niña a pensar en carreras y en las habilidades de la matemáticas necesarias para tener éxito. Las actividades también ensancharán la opinión de su niño/niña acerca de quién está en las matemáticas e introducirán algunos modelos interesantes que estan implicados en las matemáticas.

Hay varias maneras en que usted puede ser una parte de esta experiencia para realzar la comprensión de su niño/niña:

- Visite un museo del arte con su niña/niño. Busque las matemáticas en el arte—¿utilizó el artista formas o patrones geométricos?

- Vaya en línea 4con su niña/niño a buscar muestras del arte de Escher y/o de Mondrian.

- ¡Deje a su niña/niño que le enseñe sobre las matemáticas en el arte! Pidale a ella/el que le demuestre los patrones asimétricos en Mondrian, la simetría en Escher, o cómo construir un kaleidoscopio usando una lata reciclada de papitas.

Después de varias sesiones con experiencias usando las manos y "usando la mente", el grupo del programa post-escolar irá a un museo local de la ciencia para construir una exhibición.

Venga por favor a la gran "apertura" para que veas el ArtMath de su niño/niña.

Sinceramente,

Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

Who Uses Math? Equity Activity +1: Drawing a Picture

ASM+
EQUITY
ACTIVITY

Who Uses Math?

Drawing a Picture

Before starting this activity, take some time to think about the question: Who uses math? Everyone! During the discussion students may put forward an image of a stereotypical mathematician—perhaps someone in front of a blackboard with calculus expressions on it. It is easy to inadvertently perpetuate the stereotype, so be careful!

Students should come away from this activity knowing that everyone does math and that math is a basic tool for everyday life and is needed in most jobs and careers.

1. Mount a large piece of paper on an easel, wall or blackboard and tell students they are going to collectively draw a picture. Ask them to close their eyes and picture what a person who uses math looks like—whatever picture comes to mind. Ask them to describe that person in detail and draw the picture as they describe it.
2. Ask students: Does the picture include them? Members of their families? People they know?
3. Ask, "Who else should be included in this picture?" Encourage students to include everyone—since everyone uses math! Be sure to include young/older people, people of different genders, people with/without disabilities, different races, ethnicities, anyone they can think of!

Photo by https://unsplash.com/@leeanneva on Unsplash

Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

+1 The Math Student as Artist: Creating a Kaleidoscope

+1 The Math Student as Artist: Creating a Kaleidoscope

Question
Where's the math in a kaleidoscope?

Objectives
Students will:

- Use geometry to create a kaleidoscope.
- Predict the number of images in the kaleidoscope using algebra.
- Test predictions.
- Have fun creating three-dimensional interactive art.

Where's the Math?
Students will use geometric shapes (rectangles, triangles, circles) to place a plane at a prescribed angle to create reflections in a kaleidoscope. They will measure diameter, radius and circumference. Students will predict the number of images in the kaleidoscope using algebra.

Math Skills Developed
- Drawing and constructing geometric figures
- Measuring
- Sequencing
- Using symmetry
- Using arithmetic
- Using algebra

NCTM Math Standards

Content Standards

- **Geometry:** Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes; understand relationships among the angles and the length of the sides of similar objects; explore congruence and similarity; and make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.
- **Algebra:** Understand patterns, relations, and functions; describe, extend, and make generalizations about geometric and numeric patterns; and represent and analyze patterns and functions using words, tables, and graphs.

Process Standards

- **Problem Solving:** Apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematical problem solving.
- **Communication:** Communicate mathematical thinking coherently and clearly to peers, teachers, and others; and use the language of mathematics to express mathematical ideas precisely.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

Materials
For each student:

- ☐ Stackable chip can (like Pringles), with the plastic cap
- ☐ White construction paper (21 cm X 28 cm)
- ☐ 1 sheet of transparency film to act as "mirrors" (e.g., transparency for overhead projector)
- ☐ Beads—clear and color transparent—or sheets of clear plastic in various colors
- ☐ Corrugated cardboard (any recycled carton will do)
- ☐ A template for tracing and cutting the parts of the kaleidoscope (for students who are visually impaired)
- ☐ Large pieces of cardboard (as backing for art pieces)

For each small group:

- ☐ Markers
- ☐ Ruler
- ☐ Compass
- ☐ Protractor
- ☐ Clear tape
- ☐ Scissors
- ☐ Permanent markers
- ☐ Paint
- ☐ Glue stick
- ☐ Can opener
- ☐ Glitter
- ☐ Colored paper
- ☐ Colored crepe paper (for students who are visually impaired
- ☐ Masking tape in various colors to decorate the kaleidoscope
- ☐ Sir David Brewster biography

Handout
- ☐ Alma Woodsey Thomas Artist Biography

Getting Ready

Constructing a kaleidoscope is a lot of fun, but it requires some preparation. Save enough stackable chip cans (like Pringles) so that every student will have one or ask each student to bring one from home. Check creative reuse centers and office or art supply stores for the other materials. **Construct two kaleidoscopes in advance as demonstration models.** One kaleidoscope should have three mirrored surfaces and one should have two mirrored surfaces. The second kaleidoscope will be used by the students for comparison later. Watch this video and again before leading the students in building theirs.

For students with visual impairments, particularly those with low vision, create a large mosaic-style drawing or tactile representation of the view inside the kaleidoscope. Using colored crepe paper cut in circles and triangles can give a big, bright impression of what is in the kaleidoscope.

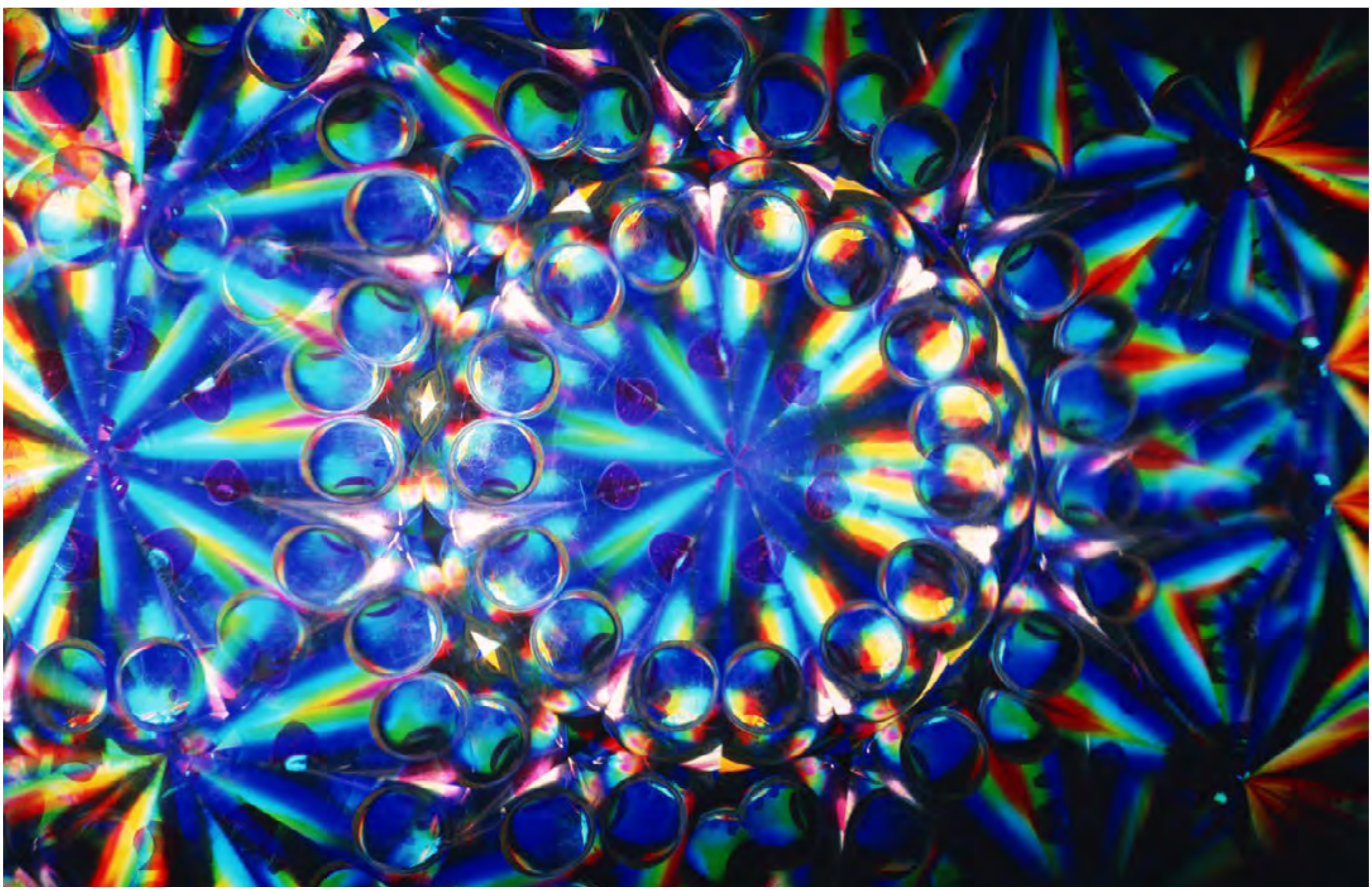
In addition, place beads in a Petri dish (a clear round container named after a 19th century German scientist) or another kind of clear container. When you place this container on an overhead projector, students will be able to observe changes in patterns though they will not see the reflective nature of the kaleidoscopes.

Be sure to allow plenty of time for the activity because the measurements of the kaleidoscope parts must be exact to achieve the best results. You may want to schedule the activity over several days.

For younger students, or if time is limited, it would be best to pre-cut the strips of transparency film. The strips can be used as a tracing template for the white construction paper strips. Cutting open the ends of the stackable chip cans will also save time.

A helpful strategy for students who have visual impairments or learning disabilities is to mark the sides of the tracing template with the measurements, using a permanent marker or raised glue dots. This reinforces the measurements, even though the students may not be doing the actual measuring. Also, measuring the template is a good way for students to practice before measuring their own strips.

In preparation for the identity activity, consider searching online for additional examples of art by Alma Woodsey Thomas to present on a projector while students are discussing her biography handout.



Canva

Part One: Large-Group Discussion (15 minutes)

1. Ask students if they have ever seen or used a kaleidoscope. Can they describe what a kaleidoscope looks like? Does anyone think that there is math in a kaleidoscope?
   - Write down their ideas on chart paper.
2. Show them the three-mirrored demonstration kaleidoscope you have made. Pass it around so each student has an opportunity to look into it and ask each one to describe what they see.
   - Also, be sure that sighted students are given adequate time to describe in detail what they are seeing; this provides a context for visually impaired students.
3. While students are passing the kaleidoscope around, draw the attention of the other students to the large mosaic or the overhead projector with the Petri dish on it. Ask students to compare the shapes and patterns on these to what they've seen in the kaleidoscope. What is similar? What is different?
4. Explain that each student will be making a kaleidoscope, working in small groups to help each other.
5. Share a copy of Sir David Brewster's biography (below) and ask the students if they've ever thought of a kaleidoscope as a **math** toy.

Part Two: Creating the Components of the Kaleidoscope (30 minutes)

Have students work in small groups at tables and demonstrate each step for them before they proceed. If some students need help understanding the instructions, they can take apart and examine the demonstration kaleidoscope you have made. Also see illustration on pg. 17.

1. Prepare the can.
   - Use a can opener to cut the end off the can so that it is open on both ends. Save the plastic cap from the other end.
   - Being careful with the metal edge, clean the inside surface of the can so there is no residue inside. A damp paper towel is helpful.
2. Cut strips.

   Note to Group Leaders:
   Younger children may need help with this step.

   - Cut the clear plastic transparency film into strips that measure 6.5 cm x 20 cm (point out that the three strips are congruent, that is, they coincide exactly when superimposed).
   - Cut the white construction paper into 3 strips slightly larger than the clear plastic transparency strips. If you have students in

Sir David Brewster
Scientist and Inventor of the Kaleidoscope

The kaleidoscope was invented in 1816 by Sir David Brewster. Brewster had many hobbies and interests. He was interested in mathematics, science, religion, philosophy, education, optics, photography, writing, inventions, and whether there is life on other planets.

Brewster was born in Jedburgh, a small town in Scotland in 1781. Even as a child he liked learning and was always interested in science. Because he was a good student, his family encouraged him to study for the ministry of the Church of Scotland and he attended the university in Edinburgh. But the ministry wasn't his calling—he was too interested in science!

In 1816 Brewster was only 35 years old and he was already a well-known philosopher, writer, scientist, and inventor. He learned about the use of mirrors and reflected images used in ancient Egypt to create patterns. He used these ideas and his knowledge of optics to design the first kaleidoscope. The kaleidoscope became so popular that Peter M. Roget (who created Roget's Thesaurus) wrote about him: "In the memory of man, no invention, and no work, whether addressed to the imagination or to the understanding, ever produced such an effect."

Brewster's scientific discoveries were some of the greatest of his time. He wrote two books on the kaleidoscope and several others on scientific topics.

[Read more about David Brewster.]

your group who are visually impaired, one strategy is to ask sighted students to measure and mark the paper by folding it (it will crease) and then ask visually impaired students to do the cutting. Or, visually impaired students can cut strips using a template as their guide.

- Tape the plastic onto the construction paper so that each one of the three strips has a “mirror finish” on one side (Use tape only at the ends of the strips).
- Cut the corrugated cardboard into a strip that is 1 cm x 28 cm.

3. Trace and cut circles.

- Trace (using the bottom of the can) and cut out 1 circle from the white construction paper and 2 circles from the clear plastic transparency film. (The second plastic circle will be used later on and placed at the end of the can with the cap).

4. Cut out a hole at the center of the construction paper circle.

- Draw a line across the widest part of the construction paper circle (this is the diameter). Find and mark the midpoint by measuring the line and dividing the distance by 2 (this is the radius).
- Find and mark the midpoint by measuring the line and dividing the distance by 2 (this is the radius).
- Measure and mark 1 cm from the midpoint. See figure 1 below.
- Place the point of the compass on the midpoint and the pencil end on the 1 cm mark.
- Trace a circle from this position (this will create a circle with the radius of 2 cm). See figure 2 below.
- Carefully cut the circle from the center.

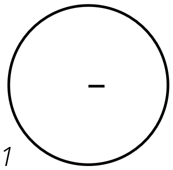


figure 1

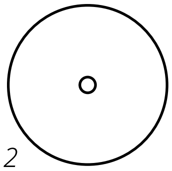


figure 2

5. Cut colored plastic into pieces smaller than 3–4 cm or use color beads and distribute to each student.

6. Beads are available in different shapes as well; providing a variety of shapes not only adds to the visual interest of the kaleidoscope but allows students who are color blind or visually impaired to express their creativity. Each student should have 25–35 small beads or pieces of plastic for the kaleidoscope.

Canva

Part Three: Assembling the Kaleidoscope (30 minutes)

1. Set the tube aside while the body of the kaleidoscope is assembled.

2. Tape the three white construction paper strips together (transparent plastic side in) to form a triangular tube.

   - Ask students to note that all of the angles of the triangle are the same (they are 60°). Use the protractor to confirm the angles. Like rulers, protractors can be made tactile using raised glue dots at every 10° mark. Do they know the name for this kind of triangle? (It is an equilateral triangle.)

3. Place a small piece of clear tape sticking upright along the edge of the white construction paper circle. Place the clear plastic circle on top and fold the tape into place.

4. Place the joined circles at the end of the can without the cap. Tape into place with the plastic circle facing the outside.

5. Slide the triangular tube inside the chip can (it should fit snugly without bending any of the sides of the triangle).

6. At the end of the can with the cap: Place one of the clear plastic circles on top of the triangular tube in the can and tape into place. Put glue on one side of the strip of corrugated cardboard and line the inside edge of the top of the can with it. Trim excess cardboard. (This will hold the plastic circle in place.)

7. Place color transparent beads (or small pieces of transparent color plastic) in the space created by the plastic circle and snap on the lid.

8. Decorate the outside of the can.

For Older Students:
Calculate the exact length of the cardboard strip needed to line the inside edge of the can. The length of the cardboard strip is the length of the circumference of the can.

Circumference = $2\pi r$
(where $\pi = 3.14$ and r = radius of the circle).

Steps
- Measure the radius of the circle (distance from the outside of the circle to the center).
- Multiply by 2 to get the diameter
- Multiply by 3.14 (π) to get the circumference.

Part Four: Exploring the Kaleidoscopes (20 minutes)

! *Note to Group Leaders:*
Have your second kaleidoscope with only two mirrored surfaces available for students to use for comparison.

1. Give students time to explore their kaleidoscopes and share them with each other. Ask students to think about the activity. Was the exact placement of the plastic strip "mirrors" important? How did the placement of the mirrors affect the images?

2. Ask students if they have ever seen a kaleidoscope that had a different kind of image. How was the image different? Point out that many kaleidoscopes only use two mirrors to create patterns.

3. Pass around the two-mirrored kaleidoscope. Ask students to describe the difference between their three-mirrored kaleidoscopes and the two mirrored one. How does adding the third mirror affect the kaleidoscope? (The third mirror creates infinite repetition in the plane—a tessellation. While use of two mirrors generates a circular design, the third mirror is required to create an infinite design.)

   - The math and science of reflection can be conveyed to students with visual and learning disabilities as follows:

   » Give students a tennis ball and have them roll it on a desk or table to a flat, vertical surface (such as a book standing on edge). Allow them to experience the return of the ball in a straight line.

- » Then set up two books at an angle (approximately 60°) and have them roll towards one of the books (assisting with aim as needed). The students will experience the circular movement of the ball from one to the other and then to the side.
- » Finally, set up three books in an equilateral triangle and allow the student to roll the ball toward one book (a good strong push is helpful) and allow them to hear the multiple impacts as the simulated "reflection" hits the different sides.

4. Ask students if the images look like anything they've seen before. Ask students to describe the image the kaleidoscope creates. Are the images congruent? Are they symmetrical?

5. Revisit the chart paper with the ideas that students had about math in kaleidoscopes before the activity. Ask them, now that they have created a kaleidoscope, what math do they think they used? Add their ideas to the original chart.

Part Five: Literacy and Math Identity Activity (20 minutes)

1. Give students a copy of the Alma Woodsey Thomas biography handout. Have students read the handout aloud.

2. Have them do a think-pair-share activity with the following questions: (If you found additional examples of her art to share, project these examples during their think-pair time.)

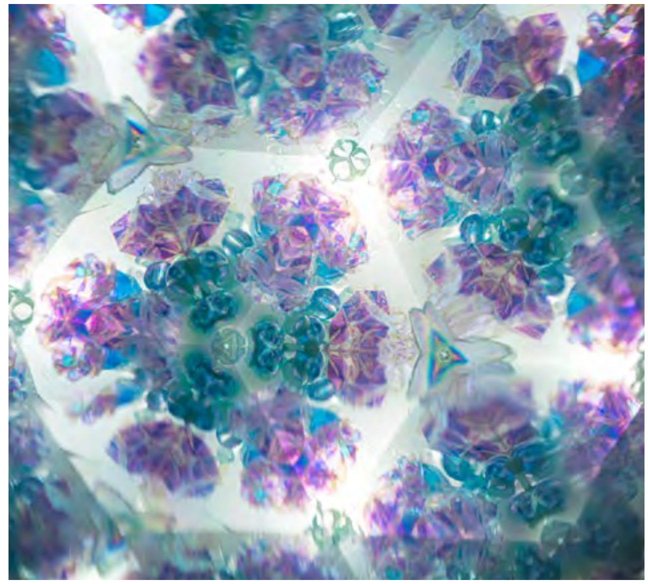
    - What makes her a role model?
    - How are they like this artist?
    - What do they see in Thomas's art on the handout that is like or different from the patterns they observed in their kaleidoscopes?

3. Ask each pair to share their responses.

Jessica Scranton for FHI 360.

Part Six: Making Art Inspired by Kaleidoscopes and Alma Woodsey Thomas (30 minutes)

Tell students they will now be making their first piece of ArtMath. They will find a view in their kaleidoscope they like and will then use that pattern and Alma Woodsey Thomas's art as inspiration for their own geometrical pattern.



Canva

1. Have students decide on a view in their kaleidoscopes that they particularly like.
2. If possible, assist the students by taking a photo through the viewport so they have a stationary image to work from as they create their first piece of art.
3. Provide students with large cardboard backing pieces, paint, markers, construction paper, glue, and scissors.
4. Tell students to paint or construct (using construction paper) a mosaic that looks like either the view from their kaleidoscope or is inspired by one of the paintings they saw from Alma Woodsey Thomas.
5. When they're happy with their pieces, ask them to name and sign their art. Hold these pieces for the culminating art show.

Additional Literacy and Math Identity Activities

If you think one of these activities would better suit your students and/or the goals of your program, you can replace Part Five with one of these activities or include these activities in addition to the one listed in Part Five.

- As a group activity, ask students to write directions for creating a kaleidoscope. Have them compare their directions with another student's to see if they are the same. Encourage them to suggest additions and agree on changes to have the most complete instructions. Point out that working collaboratively helps everyone!
- Search online for artist-made or collectible kaleidoscopes to see the variety of types and decorations.
- Search online for local kaleidoscope clubs.
- Find a copy of *Math Skills Made Fun: Kaleidoscope Math* by Cindi Mitchell (Scholastic Teaching Resources, 2003) at the local library or book store. Try one of the activities!

Equity

While some students might have good math skills and others might have good art skills, all students can be successful and students of all genders have an opportunity to be creative using the inexpensive, easily accessible materials in this activity. In addition, there are many ways to make this activity accessible for students with visual impairments or learning disabilities.

Reflecting on the Activity

- Did all the students have fun creating the kaleidoscopes? If the measurements weren't perfect, the kaleidoscopes produced a slightly different but still pretty, image—were there students who had this kind of kaleidoscope?
- How did students feel about their work?
- Did students use math vocabulary when describing the components and assembly of the kaleidoscope?
- Did I schedule enough time?

Discussion Questions for Older Students:

1. 1) Is it possible to predict the number of images in the two-mirrored kaleidoscope?

1. Steps

   - Ask what they think will happen if the angle between the mirrored surfaces in their kaleidoscope is changed. (The number of images changes!)
   - Tell the students that it is possible to predict the number of images in the kaleidoscope based on the angle of the mirrors. The formula for predicting the number of images is:

     $$\frac{360°}{\text{angle of mirrors}} \text{ minus } 1 = \text{\# of images}$$

     In this case, since all the angles of the triangle formed by the strips are equal, the angle is 60 degrees. So, 360 divided by 60 = 6, minus 1 = 5 images. It is interesting to note that the same formula works for looking at oneself in the mirror. 360 degrees divided by 180 degrees (flat surface) = 2, minus 1 = 1 reflection in the mirror!
   - Ask students to predict how many images there would be if a right-angle triangle (that is, a 90-degree angle) was used.
     (Using the formula: 360 degrees divided by 90 degrees = 4, minus 1 = 3 images.)
   - How many images do the students see in a three-mirrored kaleidoscope?

1. 2) Is it possible to find out what the angle in a two-mirrored kaleidoscope is by counting the images?

1. Steps

   - Count the number of images in a kaleidoscope. (Remind students that there is one true image and the others are reflections.)
   - Recall the formula for the number of reflections in a kaleidoscope.

     $$\frac{360°}{\text{angle of mirrors}} \text{ minus } 1 = \text{\# of images}$$

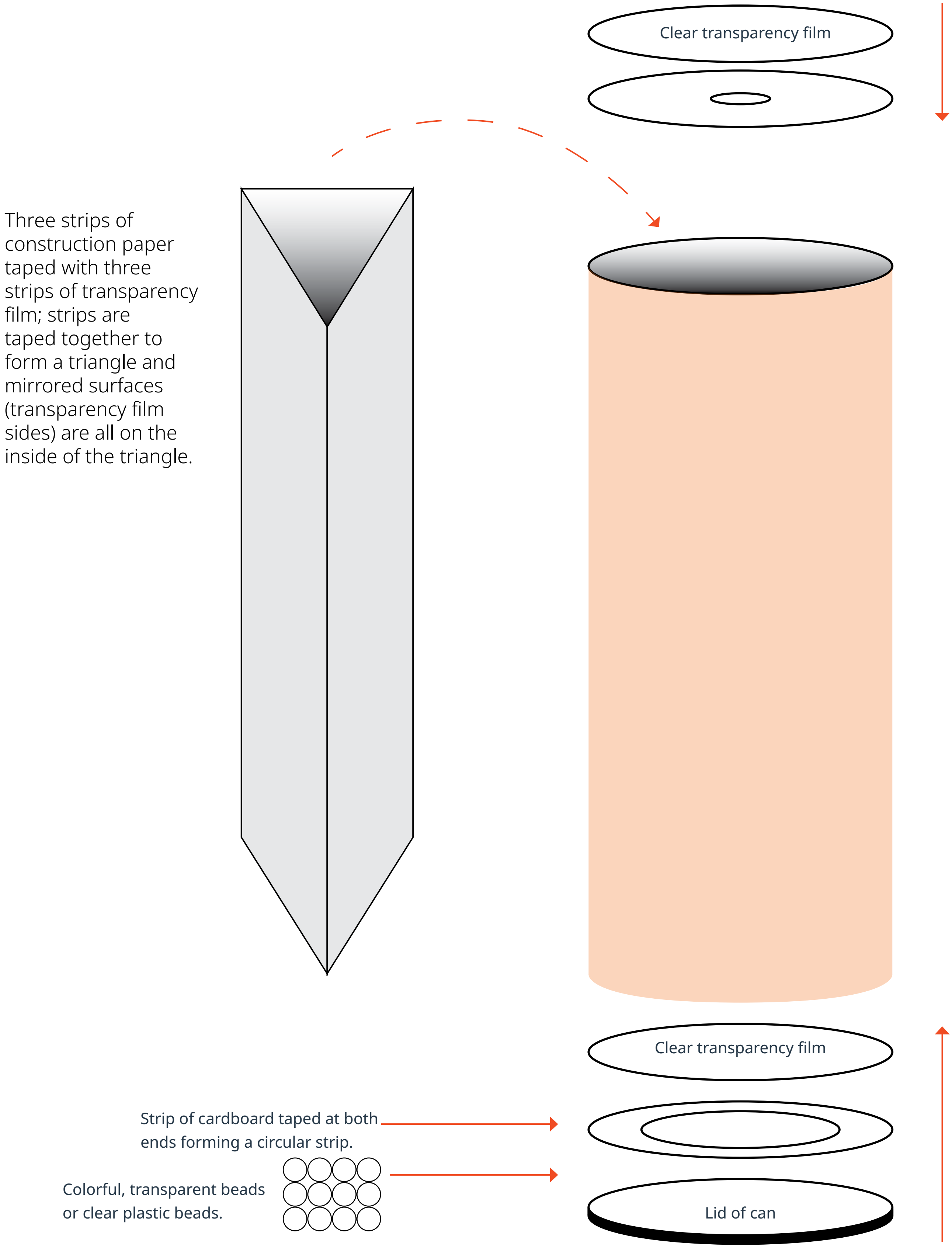
   - Rename the angle "X"
   - Plug in the numbers and symbols that are known. For example, if students counted nine (9) reflections, then:
     $^{360}/_{X} - 1 = 9$ (solve the equation)

     1. $^{360}/_{X} - 1 = 9$ (add 1 to both sides)
        $+ 1 \quad + 1$

        $^{360}/_{X} = 10$ (multiply both sides by X)
        $\times X \quad \times X$

        $360 = 10X$ (divide both sides by 10) SOLUTION: X is a 36° angle
        $10 \quad 10$

Creating Your Kaleidoscope



Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

+2 M.C. Escher Having Fun with Geometry

+2 M.C. Escher Having Fun with Geometry

ASM+ INSTRUCTOR GUIDE

Question

How can we use geometry to have fun and create art in Maurits Cornelis (M.C.) Escher's style?

Objectives

Students will:

- Create a tessellation using geometric figures.
- Explain concepts using mathematical terminology as they discuss their art with each other.
- Have fun creating art.
- Learn about M.C. Escher and Doris Schattschneider.

Where's the Math?

A tessellation is a repeating pattern of figures that covers a plane (flat surface) without any gaps or overlaps. Tessellations appear in nature—the construction of a honeycomb in a beehive and the way some crystals are formed are tessellations. They help students understand the concept of symmetry. Tessellations can be a representation of any of the following symmetries:

- **Translation**—repetition along a line
- **Reflection**—repetition along an axis
- **Rotation**—repetition around a point
- **Glide reflection**—a combination of translation and reflection.

(See handout at the end of this theme for a graphic depiction of these terms.)

Math Skills Developed

- Measurement
- Identifying geometric figures
- Creating patterns using geometric figures
- Understanding and recognizing tessellations

Materials

- ☐ Paper
- ☐ Sticky notes (e.g., 3x3 inch) of assorted colors
- ☐ Pencils
- ☐ Markers
- ☐ Crayons
- ☐ Paints and brushes
- ☐ Rulers
- ☐ Protractors
- ☐ Scissors
- ☐ Glue
- ☐ Student kaleidoscopes
- ☐ Pipe cleaners
- ☐ Sandpaper (different levels of coarseness)
- ☐ Doris Schattschneider mathematician biography

Handouts:

- ☐ Examples of Tessellations and Tessellated Patterns
- ☐ Transformations and Symmetry
- ☐ M.C. Escher artist biography (below)

NCTM Math Standards

Content Standards

- **Geometry:** Think about the results of subdividing, combining, and transforming shapes; and explore congruence and similarity.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; and apply and adapt a variety of appropriate strategies to solve problems.
- **Communication:** Organize and consolidate mathematical thinking through communication; communicate mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

Getting Ready

Practice making a tessellation (see Step 2 for instructions)—this will be helpful for the demonstration. Make enough copies of the handouts for every student. For additional samples of M.C. Escher's work, go to mcescher.com, the official website of the M.C. Escher Foundation and The M.C. Escher Company.

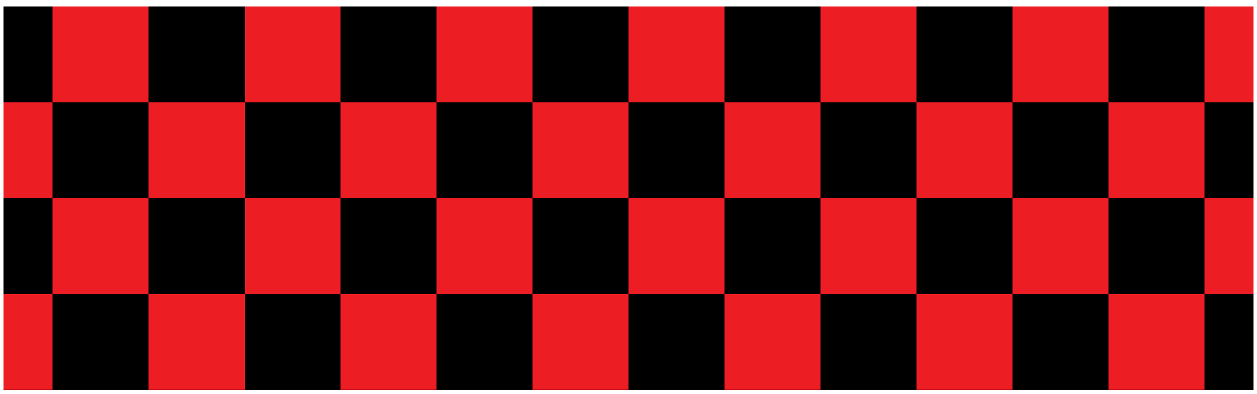
Tessellations can be made tactile in many ways. Materials, such as wooden or plastic tangrams, can be placed in different patterns. You may have some magnetic plastic tiles in different geometric shapes that you can use to build these patterns. Additionally, different textures can easily be created by using sandpaper of different levels of coarseness to represent the different colors. Tactile tessellations can be helpful for many students with different learning styles. Consider making a few example tessellations before starting this activity, especially if you have students with visual impairments who will require one of these tactile examples to complete it.

Part One: Large-Group Discussion (15 minutes)

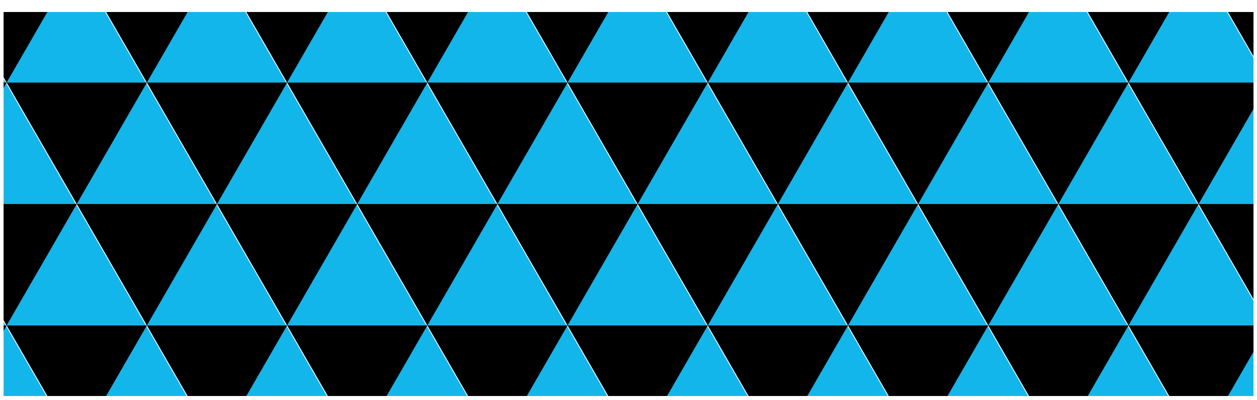
1. Review examples of Escher's work and ask students to describe it in their own words. Write down their descriptions. Students will likely say that Escher's work uses patterns (they might say "repeating" patterns). Ask students if they see any symmetry in the art. Ask if anyone knows what this special symmetrical pattern is called. (It is tessellation.)

2. Demonstrate how to make a simple tessellation using colored paper: Choose one shape and two colors. For example, choose red and black squares for younger children, and equilateral triangles or hexagons for older students. Cut enough pieces to fill a piece of paper (the number of shapes will depend on the size of the paper). Using coarse and fine sandpapers instead of colors works well for tactile learners or students with visual impairments. Be sure that the demonstration tessellation you create is large enough for all the students to see.

- **For squares:** Starting at the center of the paper, glue one red square followed by one black square and repeat until the entire paper is covered with squares. (Be sure to alternate the position of the different colors!) The result will be a pattern that looks like a checkerboard, which many students will recognize.



- **For triangles:** Starting at the center of the paper, glue a whole row of one- color equilateral triangles with the points on top. Glue the next row using the other color with the points facing down but with sides adjacent. Repeat the rows until the entire page is filled.



- Some squares or triangles will hang off the edge of the paper. These can be trimmed for the purpose of creating art, but students should be reminded that true tessellations go on forever!

3. Ask students, "Can anyone think of a different way to arrange the squares (or triangles or hexagons) that would still be a tessellation?" "Is the symmetry still there?" Students may think of putting two or three red squares together, followed by one black, and repeating that until all the squares are used. The triangle tessellation might be two red triangles up followed by one black one, and so on. It is important to keep repeating the patterns until the entire page is filled. The possible variations are what make this type of art so intriguing!

4. Ask students where they have seen tessellations in their community, home or school. Encourage them to think about other places such as tiled bathroom walls, honeycombs, school hallway floors. An example of a tessellation is the pattern of tiles on the walls of many public spaces. Other examples are sidewalks are made of hexagonal bricks. Carpets in office buildings or schools are often made of repeating symmetrical patterns that are tessellations. Ask students to look in their kaleidoscopes (from Activity 1) and describe what they see. Do they see tessellations?

5. Distribute the M.C. Escher Artist Biography, Transformations and Symmetry, and Examples of Tessellations and Tessellated Patterns handouts.

   - In a computer lab or using a computer with Internet access and a projector, help students explore the official website of the M.C. Escher Foundation and The M.C. Escher Company to view more of these tessellations.

   - Show them a video on Escher's work as examples of both math and art like those listed here so they can see additional samples of his work:

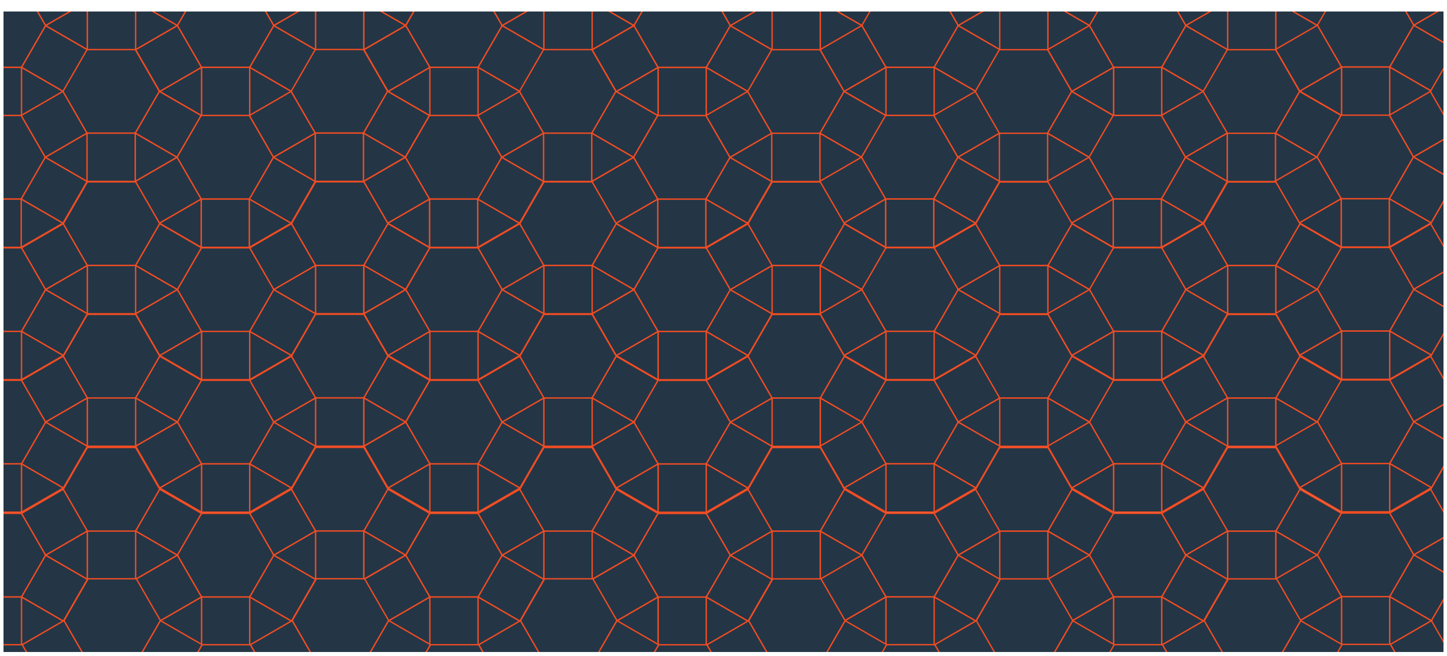
» The Mathematical Art Of M.C. Escher - YouTube
» M.C. Escher: Sky and Water 1 - Animation and Cartoon Videos - YouTube

   - Ask students to react to the video. What examples of symmetry did they notice? What examples of reflection? How are the shapes they've created like what they saw from Escher?

Note to Group Leaders:
Point out that Escher's patterns are not all the same, that he transforms his figures on the paper by using translation, reflection, rotation, and glide reflection. Use the Transformations and Symmetry handout to explain each concept. Pipe cleaners bent in the shape of an "R" or a "P" and rotated provide a concrete way to demonstrate the transformations.

6. Explain to students that they will be creating their own Escher-like work. Have them read Escher's biography and study the examples of Escher's work. Ask how it is similar to the demonstration tessellation.

7. Ask students to think about how they will use tessellations to create art. Encourage students to use one (or more) of the symmetry transformations.



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ACTIVITY

Part Two: Small-Group/Individual Activities (30 minutes)

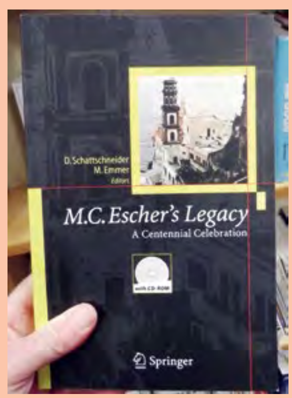
1. Ask small groups of students to create a figure or shape for their tessellation and to choose which transformation(s) they will use (translation, reflection, rotation, and glide reflection). Or, students can work on individual tessellations.
    - Allow students who are visually impaired to create a tactile figure/shape if they wish using pipe cleaners. As mentioned above, these tactile shapes can be easily rotated and repeated.
2. After selecting which transformation they want to use, ask students to draw the figure on paper to create a repeating pattern. Alternatively, give the students the option to create their art from sticky notes or other objects to create tessellations and/or repeating patterns.
3. When the pattern is complete and the entire paper is covered, ask students to add color using paint, crayons, or markers.
4. Students can think of names for their pictures and sign their work.
5. Ask students to compare their artwork (and Escher's) to the images they saw in their kaleidoscopes. Do they look similar? Are they different? In what ways? Write down students' responses on chart paper.

Doris Schattschneider
Mathematician

Symmetry and geometric models have always been fascinating for Doris Schattschneider. Her interest in both geometry and art led naturally to the study of tiling, and the work of the Dutch artist M.C. Escher. She is internationally known for her work with tessellations of the plane and her mathematical thinking on M.C. Escher's art.

Schattschneider holds a Ph.D. in mathematics from Yale University, and is currently professor emerita of mathematics (retired 2002) at Moravian University. In 1993 she received the national Mathematical Association of America Award for Distinguished Teaching of College or University Mathematics.

Schattschneider has written many articles and books about mathematics, including math activity books. She was the "Geometer" on the Visual Geometry Project, which produced *The Geometer's Sketchpad*, along with videos and activity books on polyhedrons and symmetry.



"M.C.Escher's Legacy" by brewbooks on flickr, used under CC BY-SA 2.0 license, cropped.

Part Four: Sharing Findings

(30 minutes)

Display all the students' work. Ask students to observe each other's work and identify the shapes and transformation(s) used. Ask each student to make a presentation to the whole group that includes how the artwork was created, which shapes and transformations were used, and why the shape or figure was chosen.

Part Five: Literacy and Math Identity Activity (15 minutes)

Have the students read Doris Schattschneider's biography:

1. Ask how surprised they were to think of shapes and symmetry as math.
2. Have students write their own brief biography on themselves as mathematicians (point out that they just used math to create art).

Additional Literacy and Math Identity Activities

If you think one of these activities would better suit your students and/or the goals of your program, you can replace Part Five with one of these activities or include these activities in addition to the one listed in Part Five.

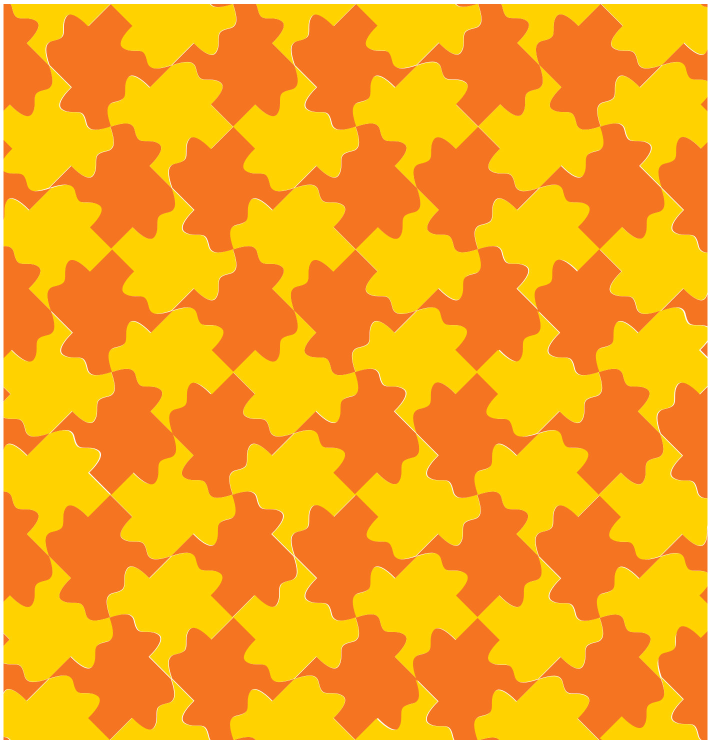
- Use the Escher Coloring Book (see resources) to look for patterns and transformations. Ask students to label the pictures according to the transformations used.
- Help students use a free online graphic design application to create animated versions of their tessellations like in the videos they watched. Have groups of students work together to create the animation and to add voiceover narration explaining the mathematical concepts they used.
- Have students work in pairs to create a presentation where they each present their tessellations, explain the math of tessellations, and then compare their individual designs. Have students present to other groups at the afterschool center to practice before the culminating art show.

Equity

Some students may feel less artistic than others and therefore not as willing to participate in the presentation process. Remind students that all their art is valuable because it is an expression of their thoughts.

Reflecting on the Activity

- Did the students feel good about the art they created?
- Were they able to describe their process for using the geometric transformations?
- Were they able to identify the math in their artwork?
- Were they comfortable speaking before a group?
- What could I do to make them feel more comfortable?



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Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

+3 The Pattern Tessellation Game

+3 The Pattern Tessellation Game

ASM+ INSTRUCTOR GUIDE

Question
How can we tell the difference between a pattern and a tessellation?

Objectives
Students will:

- Look at several pictures of patterns and tessellations.
- Explain the differences between patterns and tessellations.
- Have fun playing an ArtMath game!

Where's the Math?
Is it a pattern or a tessellation? A tessellation is a repeating pattern of figures that covers a plane (flat surface) without any gaps or overlaps. Students have looked at and created both patterns and tessellations when they made kaleidoscopes and created art in the style of M.C. Escher. Students will observe the mathematical aspect of patterns and tessellations created by people and found in nature and categorize them accordingly.

Math Skills Developed
- Identifying patterns
- Identifying tessellations
- Identifying symmetry

Materials
- ☐ Pencils or pens
- ☐ Wax-covered yarn
- ☐ Fine and coarse sandpaper

Handouts:
- ☐ Pattern Tessellation Game sheet (one per group)
- ☐ Pattern Tessellation Samples

Getting Ready
Make copies of the Pattern/Tessellation Game sheet (one per group). Put Velcro or tape on the back of each Pattern/Tessellation picture so that they can be displayed around the room. Create two signs, one with the heading "patterns" and one with "tessellations."

Review the images in the game ahead of time to be familiar with them. If needed, create more inclusive versions of the images using the strategies listed in the two prior art activities (e.g., fine and coarse sandpaper, wax-covered yarn, glue lines) or by increasing their size.

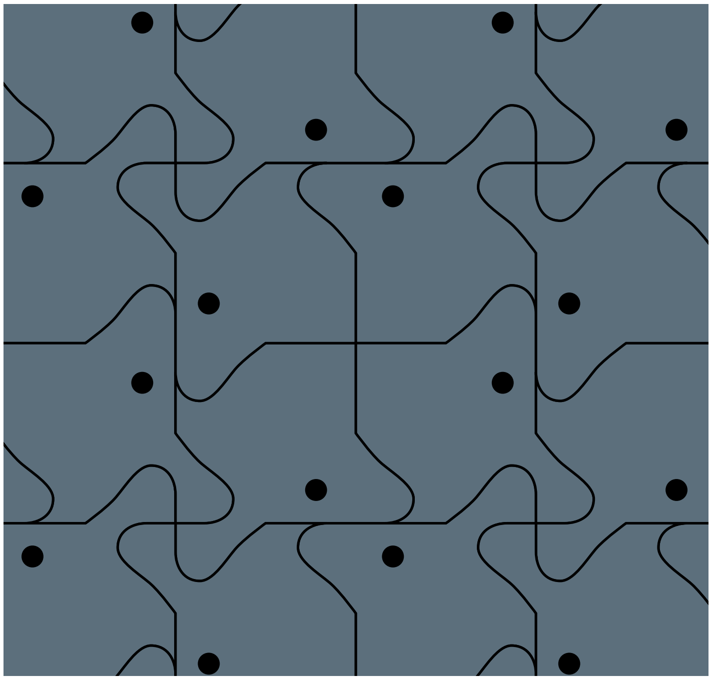
NCTM Math Standards

Content Standards

- **Geometry:** Reason about the results of subdividing, combining, and transforming shapes; and explore congruence and similarity.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; and apply and adapt a variety of appropriate strategies to solve problems.
- **Communication:** Organize and consolidate mathematical thinking through communication; communicate mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.



This activity is open-ended, so keep in mind that students' answers may vary, as there may not be only one right answer. The important thing is for students to have reasonable justification for their answers.

ASM+
ACTIVITY

Part One: Large-Group Discussion (10 minutes)

Using the kaleidoscopes, Thomas-like and Escher-like art created by students, review the concepts of pattern and tessellation. A **pattern** is a repetitive set of objects or numbers. A **tessellation** is a repeating pattern of figures that covers a plane without gaps or overlaps and has no borders or boundaries. A **regular tessellation** remains visually the same when rotated. With students, make a list of the characteristics of patterns and tessellations. Post the chart in the classroom.

Part Two: Literacy and Math Identity Activity (10 minutes)

1. Ask students to think about times when they've noticed a pattern. Make a list of these times on chart paper or the board. If students mostly focus on visual patterns, encourage them to consider other kinds of patterns. How many of these patterns might include math?

2. Now ask students to think about what kinds of careers the adults in their lives have—is understanding or using patterns important to any of these careers? What other careers can they think of where understanding or using patterns would be important?

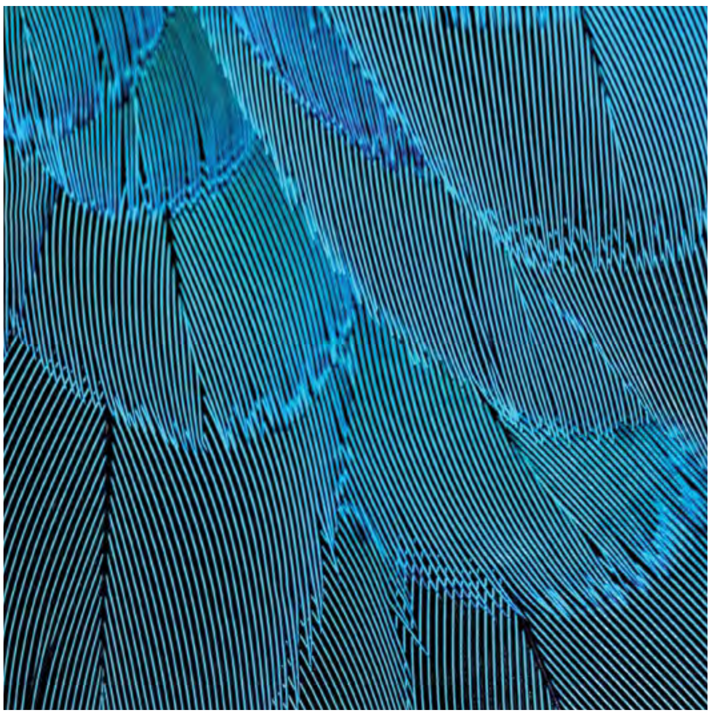


Photo by https://unsplash.com/@davidclode on Unsplash

Photo by https://unsplash.com/@enginakyurt on Unsplash

Part Three: Small-Group Activity (20 minutes)

Tell students they will be practicing recognizing patterns and tessellations together in a game.

1. Divide students into small groups and give each group a copy of the game sheet, which has numbered patterns and tessellations.

2. Ask students to work as a group to complete the game sheet. Encourage everyone in the group to participate by providing justifications for their choices.

Part Three: Sharing Findings (15 minutes)

Review the pictures. Ask students to help organize them under each sign according to patterns or tessellations. Ask students to explain to the group why the pictures should be under patterns or tessellations. (Every picture has a pattern, only some are tessellated.)

If there are students who are blind or have visual impairments in the group, ask students to describe the art in detail. For example, “This drawing is a pattern because it uses three triangles in a row followed by a rectangle and then repeats the same sequence over again.” This articulation of the details will be helpful to all students. This strategy benefits everyone in the room—the youth who hear the description and the youth who use math as a language to describe it.

Additional Literacy and Math Identity Activities

If one of these activities better suits your students or the goals of your afterschool program, replace Part Two with it. Or add these activities into the math to provide extra math-identity building.

Have the students:

- Write directions to play the game. Have them compare their work with another student to see if they are the same. Encourage them to suggest additions and agree on changes to have the most complete instructions. Point out that working collaboratively helps everyone!
- Have a discussion about role models. What important characteristics should a positive role model have?
- Search online to find other pictures to add to the game. Help students as needed.
- Make a list of tessellations in nature as a group. (Hint: There were some examples in the game!)

Equity

Even students who do not think of themselves as artistic can enjoy this game! Remind students that all their ideas are important and that they were all able to play the game because they are all good at math.

Reflecting on the Activity

- Did the students have fun using math concepts to play a game?
- Were they able to describe the characteristics of tessellations and patterns?
- Did they recognize the concepts in their own art?

Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

+4 Piet Mondrian Creating Intriguing Art

+4 Piet Mondrian Creating Intriguing Art

ASM+ INSTRUCTOR GUIDE

Question
How can art help us understand unique numbers like phi (golden ratio), geometry, and asymmetry?

Objectives
Students will:

- Learn about Piet Mondrian and his use of rectangles, lines, and asymmetry to create art.
- Learn about the golden rectangle and golden ratio.
- Use geometry, arithmetic, and measurement to create art.

Where's the Math?
Students will learn to identify and draw geometric shapes, asymmetry, and the golden ratio. Using mathematics, they will combine regular and irregular geometric shapes, measurement, symmetry, asymmetry, and color to create art. Understanding symmetry and asymmetry is important in mathematics. Asymmetry is more than just the absence of symmetry. It is an affirmation that the elements of an object are discrete and unique—that its mathematical function and format are unique.

Math Skills Developed
- Measuring
- Sequencing
- Understanding and using asymmetry
- Using geometry

Materials
- ☐ Paper
- ☐ Pencils
- ☐ Crayons
- ☐ Sand, glitter, cardboard
- ☐ Wax-covered yarn
- ☐ Markers
- ☐ Paint, paint brushes, and water
- ☐ Rulers
- ☐ Examples of Mondrian's art

Handouts
- ☐ Geometric Shapes
- ☐ Piet Mondrian Artist Biography

Getting Ready
Search the public library or open-source and Creative Commons image sites on the internet to find copies of Piet Mondrian's art (be sure to include Broadway Boogie Woogie). Several of his works, including Broadway Boogie Woogie, are included on the Piet Mondrian Artist Biography handout below, but we encourage you to find more examples for your students. Openverse is an easy-to-search site where all the content is either under a Creative Commons license or is in the public domain; most government sites, like those of the Smithsonian Institution, also have images available through Creative Commons or public domain licensing, especially for educational purposes. Many of Mondrian's works are in the public domain in the U.S., so you may also find some on Wikimedia Commons, though the citation data there may not be accurate. Be sure to follow the licensing directions for attribution as needed. Make copies of the art for pairs or groups of students to share, and provide the geometric shapes handout to all students. Review the definitions of geometry, symmetry, asymmetry, Neoplasticism, and art in the glossary.

Mondrian's art can easily be made tactile for students who are blind/visually impaired by using different textured items (sandpaper, glitter, cardboard, etc.) to represent the differently colored rectangles. Wax-covered yarn or gluing down string can represent the black lines.

In addition, some students who are color-blind may have difficulty recognizing the different colors in Mondrian's

NCTM Math Standards

Content Standards

- **Measurement:** Understand measurable attributes of objects and the units, systems, and processes of measurement.
- **Geometry:** Analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Process Standards

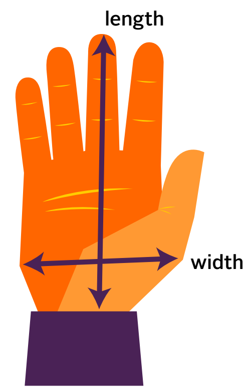
- **Problem Solving:** Solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems.
- **Communication:** Communicate mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others.
- **Connections:** Recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and recognize and apply mathematics in contexts outside of mathematics.
- **Representation:** Create and use representations to organize, record, and communicate mathematical ideas.

art. Although the most common form of color blindness expresses itself as difficulty distinguishing between reds and greens, there are many other types. During the group discussions, be sure to give verbal cues such as, "Here we see a red triangle, and then a yellow triangle," and so on. Students who are color-blind can also use crayons with the names on them so that they can match them to the squares.

Consider pre-cutting shapes of different colors or providing magnetic shapes for visually impaired students to use for this activity.

Part One: Large-Group Discussion (15 minutes)

1. Ask students to measure their hands, first from side to side and then from the top to the bottom. Ask them to divide the width into the length. Add up everyone's answer and divide by the number of students to get the average ratio in the group. The ratio of hand widths to hand lengths is (approximately) 1 to 1.6! Write the number someplace highly visible in the room and tell the students that the ratio is an important number and ask if any know why. They're about to find out! Mention that this unique number is called phi.



> *Note to Group Leaders:*
> This activity works better with a larger group (the larger the data set, the closer the result is to phi!)

2. Review a sample of Mondrian's artwork.

3. Ask students to discuss the painting and to think about how Mondrian's work reflects geometrical shapes. Chart the ways that students think geometry and math are reflected in Mondrian's work. What shapes does Mondrian use? How does he place lines to create interest?

   - Introduce the idea of a golden rectangle—a rectangle where the sides are in a ratio of 1 to 1.6. Explain to students that this is a ratio found in the world around us! (See description in the Glossary.) Students can measure the sides of rectangles in Mondrian's art and use that information to form ratios to see which figures are golden rectangles. Note to group leaders: This concept may be too difficult for young students.

   - Mention to students that the golden rectangle and golden ratio appear all the time in art and in nature. It is considered visually beautiful and is taught in art school and marketing classes! (Share examples of art and commercial use of the ratio at the end of the curriculum.)

   - Bring up the examples of Alma Woodsey Thomas's art that you showed students in the first activity. Ask them to compare Thomas and Mondrian. How are they alike and how are they different in the ways they use shapes and symmetry?

Part Two: Small-Group/Individual Investigations (30 minutes)

1. Ask students to draw an asymmetric pattern using any rectangular shape and then outline each shape in black using a marker or crayon. Students who are blind/visually impaired may prefer to use pre-cut rectangular shapes that can be pasted to paper in order to create the asymmetric pattern.

2. Ask students to plan how they will use paint, crayons, or markers in primary colors (blue, red, and yellow) to fill in some of the outlined rectangles. Will there be a pattern to the way they fill in the colors? As they are planning, remind students again that their Mondrian-style picture must be asymmetrical.

3. Have students create their Mondrian-style art using markers, crayons, colored paper, paint, wax-covered yarn, string, or a digital illustrating tool.

4. After deciding on a name for their art, students should label and sign their work.

Part Three: Sharing Findings (20 minutes)

Display the Mondrian-style art. Bring the group together and ask each student to present their work. If the students have worked in groups, each group should present its work. The presentation should include how the artwork was created. How many and what kind of shapes were used? Did students measure the distance between lines or shapes? What measuring tools did they use? Does the picture have a repeating pattern? Students should also report on how the painting got its name and any other special features that the artist(s) would like to include.

Save the artwork for display and for the ArtMath culminating event.

Part Four: Literacy and Math Identity Activity (20 minutes)

Show students a picture of Mondrian's Broadway Boogie Woogie on the website for The Museum of Modern Art (MoMA).

1. Ask them why they think Mondrian gave the painting that name.
2. Read the description from MoMA aloud or play the audio from their Kids playlist audio tour, which includes a transcript for students who are deaf or hard of hearing.
3. Show them the video of Jason Moran "playing" the painting as a piece of jazz music.
4. Ask students to write about their artwork as if the description would appear on a website with their art. (If the afterschool center has a website where the art can be featured, the description can be included.)
    - You can also provide an alternative option where students who are interested in theater, film, or video can write the script to a short video explaining how math and art are incorporated into their pieces. A potential extension of this activity could be using these scripts to shoot and edit together cell phone videos, which can be shown during the culminating gallery activity or on the center's website or social media (with caregiver permission).

Additional Literacy and Math Identity Activities

If one of the below activities better suits your students or center goals, replace Part Four with it, or use the below activities to further build your students' math identities.

- Remind students they have now read several artist biographies. Have them write their own short biography. Ask them what the most important part to include about themselves is.
- Have students make a list of careers where the golden ratio might be used (e.g., architecture, marketing).
- Have students explore online for websites that have additional work by Mondrian.
- Share one of the problem-solving activities from *Math-Terpieces: The Art of Problem-Solving*, by Greg Tang (New York: Scholastic Books, 2003). Ask students to read the instructions to each other and do some of the fun activities.

Equity

Students of all gender identities will be equally engaged in this fun activity to create art. All will gain a greater understanding of measurement and geometry content and skills. Be sure to use the suggested materials and modifications that make the activities accessible to students with disabilities.

Reflecting on the Activity

- Did the students have fun doing this activity?
- Were they able to describe their process for creating art using the "language" of math?
- Did all the students have a chance to present their art?

Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

+5 An ArtMath Field Trip

+5 An ArtMath Field Trip

Question
How does math add to the beauty of artwork?

Objectives
Students will:

- Review mathematical terms that relate to the art of Brewster, Thomas, Escher, and Mondrian.
- Collect data for research about math and art at a library, museum, or on the Internet.
- Explore the art of Escher and Mondrian firsthand.

Where's the Math?
As the students view art at the museum, library, or on the Internet, they will see applications of patterns, tessellations (including translation, reflection, rotation, and glide reflection), symmetry and asymmetry, and the golden ratio. The students collect data for their research about math and art.

Math Skills Developed
- Use of mathematical terms
- Identify math concepts in a context outside of mathematics
- Collecting data

Materials
- ☐ Computer (for a virtual field trip)
- ☐ Pens and pencils
- ☐ Notepaper
- ☐ Clipboards

NCTM Math Standards

Content Standards

- **Algebra:** Understand patterns, relations, and functions; describe, extend, and make generalizations about geometric and numeric patterns; represent and analyze patterns and functions, using words, tables, and graphs.
- **Geometry:** Identify, compare, and analyze attributes of two-dimensional shapes and develop vocabulary to describe the attributes; apply transformations and use symmetry to analyze mathematical situations; predict and describe the results of sliding, flipping, and turning two-dimensional shapes; identify and describe line and rotational symmetry in two-dimensional shapes and designs; and create and describe mental images of objects, patterns, and paths.

Process Standards

- **Problem Solving:** Solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.
- **Communication:** Use the language of mathematics to express mathematical ideas precisely.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

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Handouts

- ☐ Field Trip Viewing Sheet
- ☐ Gordon Sasaki Artist and Educator Biography

Getting Ready

Make plans for a field trip so students can see firsthand the work of Thomas, Escher, and Mondrian. Think about the different options available:

- Is there an art museum nearby that displays artwork by Thomas, Escher and/or Mondrian? Can you arrange a field trip?
- Is there a library nearby? Does it have a collection of art books that include Thomas, Escher, and/or Mondrian?
- If there is no museum or library nearby, consider a virtual field trip on the Internet.
- If you have access to a color printer, you may also be able to print art and re-create a gallery in your space.

After deciding the kind of field trip students will take, make a plan that will produce the best results. For example, if the group is going to the museum or library, call ahead.

- Can a guided tour be arranged?
- Can books on Mondrian and Escher be reserved?
- Can the librarian make suggestions?
- Check on accessibility features such as entrance ramps, elevators, descriptive audio devices, Braille or large-print handouts for museum exhibits, etc.

If the field trip is virtual, be sure that there is enough computer access for all the students. Try some of the museum sites (see the Resources list) first so that you are familiar with them.

Many accessibility options are available on both Windows and Mac operating systems. These can be helpful for students with visual, learning, and mobility disabilities. You can modify the size of fonts, zoom in on different objects, use slow keys and sticky keys—which ignore unintended keystrokes or group keystrokes, respectively—adjust brightness of the monitor, alter the manipulation of the mouse, or choose any number of other options.

Confirm plans for the field trip with the librarian, museum staff, or staff in charge of computer access at the afterschool program.

Note to Group Leaders:
For any field trip, there should be enough adult supervision to ensure the safety and well-being of all the students participating. This includes virtual field trips on the Internet as some websites that students might find may not be suitable.

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Part One: Before the Field Trip (10 minutes)

After having explored the various ways Brewster, Thomas, Escher, and Mondrian used mathematical concepts, and having created their own kaleidoscopes and Thomas-, Escher- and Mondrian-style art, students will be excited to see more work by these artists. The field trip will give students an opportunity to see for themselves the beauty in the connections between math and art. Before the field trip:

- Together, review the art in the style of Brewster, Thomas, Escher, and Mondrian that students have created.
- Review ways to find information at the library, museum, or on the Internet.
- Supply students with clipboards, several viewing sheets, pens or pencils, and notepaper so that if they see something that interests them, they can sketch it or write about it. (If the class is taking a virtual trip on the Internet, they can print out the information from websites that have works by Mondrian and Escher.)

Photo by https://unsplash.com/@namzo on Unsplash

Have fun on the trip!

Part Two: During the Field Trip (45 minutes)

Give students plenty of time to look around at the museum or library. If taking a virtual field trip, let the students search for sites with art and spend time viewing the pictures of Thomas, Escher, and Mondrian or other artists who used these mathematical concepts.

As students view artwork, ask them to answer the questions on the viewing sheet, which will help them focus on the mathematics within each piece. For example, ask them to identify the geometric shapes used. Can the students identify how the artist used the shapes? Are there tessellations? What kind of tessellations? Do students see symmetry and/or asymmetry in the art?

Ask students to record their responses or make sketches for discussion after the trip. If there are students who are blind or have visual impairments in the group, ask other students to carefully describe the art, paying special attention to the shapes and other math-related aspects. Be sure to leave time for students who are blind or visually impaired to ask questions so that they get as full a sense of the art as possible.

Part Three: After the Field Trip (20 minutes)

1. Using their viewing sheets as a reminder, ask students to share what they discovered on their trip. Ask students to talk about any new ideas they might have about the connections between art and mathematics. Write down their ideas on chart paper.
2. After viewing the artwork, ask the students how many observed translation? Reflection? Rotation? Glide reflection? Help the group to tally their responses by type. For example, make a bar graph with their responses. Can students guess which kind of tessellation was most prevalent in the art they saw?
3. After viewing the Mondrian artwork, ask the students if they observed asymmetry or golden ratio. Ask about the colors Mondrian used: Were they part of the asymmetry?
4. Ask students to reflect on all the artwork they have seen. Which ones appealed to them most? Why?

Part Four: Literacy and Math Identity Activities (20 minutes)

Distribute the Gordon Sasaki Artist and Educator Biography handout.

1. Read Gordon Sasaki's biography. Hear Gordon Sasaki discuss his artwork, his experiences living and working in New York, how the passage of the Americans with Disabilities Act in 1990 affected his life, and more in an episode of the National Endowment for the Arts (NEA) Art Works podcast. A transcript is included on the episode site for students who are deaf or hard of hearing.

2. Show students more of his work (some of which is for sale) at Saatchi Art. Preview his book of photography, NY Portraits, published in 2011, on the publisher's site, Blurb. This book of photographs collects portraits Sasaki took of artist friends in New York City with disabilities.

3. Ask the students why they think he was chosen as a role model. What important characteristics does he have?

Additional Literacy and Math Identity Activities

If one of these activities better suits your students or the goals of your afterschool program, replace Part Four with it. Or add these activities into the math to provide extra literacy practice and math identity building.

- Divide the group in thirds (or fourths if this activity is in addition to Part Four) and ask one group to write a series of questions they would ask Escher about his art and mathematics in an interview. Ask another group to write their questions for an interview with Mondrian. Ask the third group to write their questions for an interview with Thomas. (If your group did Part Four, ask the fourth group to write their questions for an interview with Sasaki.)
- Ask students to present their Thomas-, Mondrian-, and Escher-like work to another group at the center and talk about their field trip.

Equity

For some students, a visit to an art museum may be a new experience. These students may be excited and need time to look around, relax, and enjoy the museum before they can focus on the mathematical aspects of the trip. Be sure to take the time needed for every student to get the most from the experience.

Reflecting on the Activity

- Were all students engaged in the activity?
- Was enough time scheduled to look around the museum, library, or Internet?
- Did the students see all the connections between art and math?
- Did the students have questions that the group could use as follow-up to the trip?

Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

+6 Career and Role Model Connections

Career and Role Model Connections

There are so many ways in which art and math converge. David Brewster's interest in mathematics and optics led him to create the kaleidoscope. M.C. Escher created a new school of art that used many mathematical ideas, and Doris Schattschneider, a mathematician, uses Escher's art in her college math courses. Piet Mondrian also developed a new form of art called Neoplasticism. Gordon Sasaki was artist in residence and an educator at The Museum of Modern Art. Alma Woodsey Thomas was an abstract artist and art teacher.

1. **Sir David Brewster** invented the kaleidoscope in 1816. He was interested in mathematics, science, education, optics, and many other things.

2. **M.C. Escher** loved to draw from the time he was a child. After a trip to Spain, where he studied Moorish art, he became inspired by mathematical ideas and the use of shapes in geometry.

3. **Piet Mondrian** always knew he wanted to be an artist. Early in his career, Mondrian painted still lifes and landscapes. While working in the Netherlands, he refined his style by using fewer colors and geometric shapes.

4. **Gordon Sasaki** discovered his love for art at an early age. A fifth-grade teacher influenced him to continue to study art. He became an educator and taught about the connections between art and math.

5. **Doris Schattschneider** is internationally known for her work with tessellations of the plane and her mathematical thinking on M.C. Escher's art.

6. **Alma Woodsey Thomas** was an abstract artist and a teacher. In 1972, at the age of 68, she became the first African American artist to have a one-woman show at the Whitney Museum of American Art in New York City.

Make copies of the biographical information and handouts for students. Ask for volunteers to read the biographies aloud.

- Ask students to make a list of their interests. What careers come to mind if mathematics is combined with their interests? Chart their ideas.
- Art and mathematics are combined in many careers outside of the visual arts—for example, in architecture and engineering. Ask students to think about ways that math is part of other careers.
- Invite a speaker to the afterschool center to talk about how mathematics is used in their career. Just about anyone can be invited since math is used by everyone every day!
- Ask students to think about some of the skills and knowledge they are gaining now, both in school and in afterschool. How do they think they will use these skills and knowledge later in life?
- What kind of careers might use patterns or tessellations? Make a list with students.
- Choose a career from the student list and find someone who does that job to visit and talk about their career.
- Ask students to explore their many talents. What do they do well? What are they interested in? Make a list—be sure to include every student!
- Ask students if there are any related careers that interest them. Help them find information on how to prepare for that career.
- Ask students if they are surprised that there is so much math in art. Help them make a list called *Careers That Require Math*. Any career listed will be valid since virtually all careers require math.

Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

+7 Handouts

ASM+
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ArtMath Role Model Biographies

Sir David Brewster
Scientist and Inventor of the Kaleidoscope

The kaleidoscope was invented in 1816 by Sir David Brewster. Brewster had many hobbies and interests: mathematics, science, religion, philosophy, education, optics, photography, writing, inventions, and whether there is life on other planets.

Brewster was born in Jedburgh, a small town in Scotland in 1781. Even as a child he liked learning and was always interested in science. Because he was a good student, his family encouraged him to study for the ministry of the Church of Scotland and he attended the university in Edinburgh. But the ministry wasn't his calling—he was too interested in science!

In 1816 Brewster was only 35 years old and he was already a well known philosopher, writer, scientist, and inventor. He learned about the use of mirrors and reflected images that were used in ancient Egypt to create patterns. He used these ideas and his knowledge of optics to design the first kaleidoscope. The kaleidoscope became so popular that Peter M. Roget (who created *Roget's Thesaurus*) wrote about him, "In the memory of man, no invention, and no work, whether addressed to the imagination or to the understanding, ever produced such an effect."

Brewster's scientific discoveries were some of the greatest of his time. He wrote two books on the kaleidoscope and several others on scientific topics. Read more about David Brewster.

M.C. Escher
Artist

Maurits Cornelis (M.C.) Escher was born in Leeuwarden, Holland in 1898. He created a new school of art that used many mathematical ideas—especially geometry. Although he struggled in school, Escher liked to draw. When he was young his family wanted him to become an architect.

As a young man, Escher traveled to Spain and was fascinated by the tiled patterns created to decorate the Alhambra (an ancient Islamic palace in Grenada, Spain), and started creating tiled patterns in his drawings. He became inspired by mathematical ideas and the use of shapes in geometry.

Escher created art by taking geometric shapes and modifying the shapes so that they fit together like puzzle pieces to create very interesting patterns. His art became popular by the mid-1950s, and he became highly respected by mathematicians who identified the visualization of mathematical principles in his art. This was quite surprising since Escher had no formal training in math after high school! He once said of his work, "For me it remains an open question whether [this work] pertains to the realm of mathematics or to that of art."

Piet Mondrian
Artist

Piet Mondrian was born in 1872 in Amersfoort, the Netherlands. His first name was "Pieter," but he shortened it to "Piet." Mondrian always knew he wanted to be an artist. He studied at the Rijksakademie van Beeldende Kunsten in Amsterdam. Until 1908, his work was naturalistic and traditional. His paintings were mostly still-lifes and landscapes.

Mondrian studied the works of artists like George Braque and Pablo Picasso, who were experimenting with other styles and started painting in less traditional ways. When World War I broke out, Mondrian was in the Netherlands. It was there that he refined his style, by using fewer colors and geometric shapes. This new style was called Neoplasticism, a school of art using formal structures and primary colors.

During World War II, Mondrian moved to New York City. There he met many American abstract artists and published essays on Neoplasticism. His work started to reflect life in New York City. In 1942, his first solo show took place in New York. One of his most famous works is Broadway Boogie Woogie, now in the collection of the Museum of Modern Art in New York City.

It is important to note that, while much of Mondrian's later art contains geometrical shapes, this does not mean that Mondrian directly used mathematics in his approach. In fact, a 1995 exhibition demonstrated that the geometry in Mondrian's paintings were arrived at through trial and error ... until a balance of lines and planes was right.

Gordon Sasaki
Artist and Educator

Art has been an integral part of Gordon Sasaki's life since childhood. In grade school, he always found the arts to be an immediate and important means of expression and communication.

In third grade, Sasaki was selected by his teacher to be the student to illustrate the image of Santa Claus on the school's Christmas mural. He felt a bit shy because of the attention,, but after completion he was proud and happy to share his abilities with his fellow classmates. This seemingly insignificant occurrence built Sasaki's sense of confidence and self-esteem.

In fifth grade, Sasaki had a teacher who was an artist himself and allowed his class extra art sessions during the week. He talked and showed the students examples of different types of art and gave them many invaluable tips on techniques in drawing and painting. This teacher was a significant influence on Gordon Sasaki's decision to become an artist.

While studying art in college, Sasaki had a serious automobile accident that injured his spinal cord and resulted in the use of a wheelchair for basic mobility. While his life changed overnight, he still had his art.

Sasaki returned to college to graduate with high honors and earn a graduate degree in fine arts. Since then, art has enabled him to travel the world, sharing and experiencing many new things and different points of view. For several years, he lived in New York City and worked in the education department of the Museum of Modern Art where he was an artist in residence and educator. Gordon Sasaki conducted classes and tours for students of all ages on many topics, including how art and mathematics are connected and how incorporating mathematical principles (like the golden ratio and tessellations) into artwork makes the art more aesthetically pleasing.

He is still a practicing artist, currently living in his hometown of Honolulu.

Doris Schattschneider
Mathematician

Symmetry and geometric models have always been fascinating for Doris Schattschneider. Her interest in both geometry and art led naturally to the study of tiling, and the work of the Dutch artist M.C. Escher. She is internationally known for her work with tessellations of the plane and her mathematical thinking on M.C. Escher's art.

Schattschneider holds a Ph.D. in mathematics from Yale University, and is currently professor emerita of mathematics (retired 2002) at Moravian University. In 1993 she received the national Mathematical Association of America Award for Distinguished Teaching of College or University Mathematics.

Schattschneider has written many articles and books about mathematics, including math activity books. She was the "Geometer" on the Visual Geometry Project, which produced *The Geometer's Sketchpad*, along with videos and activity books on polyhedrons and symmetry.

Alma Woodsey Thomas
Artist

Alma Woodsey Thomas was born in Columbus, Georgia, in 1891 and was the oldest child in a family of four girls. Her first artworks were sculptures made from clay that she found along a riverbank near her grandfather's home. When she was 15, she and her family moved to Washington, D.C.

In high school Thomas loved taking art classes. After high school, she attended Howard University, graduating in 1924 with the first teaching degree from its fine arts department. She went on to earn a master's degree from Teachers College at Columbia University in New York.

Thomas taught art in public schools in Washington, D.C. She organized clubs, arranged art lectures, and even created art galleries in schools to develop students' interest in and love of art.

Thomas was a modernist who used abstract styles and mixed media. She was an activist for modernist art, art education for African Americans, and the needs of the young people in Washington, D.C.

Thomas continued to create art throughout her life. Her first paintings were realistic, but by 1960 she was creating modern, abstract art. In 1972, at the age of 68, Thomas became the first African American artist to have a one-woman show at the Whitney Museum of American Art in New York City.

Activity +1

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Alma Woodsey Thomas

Artist Biography

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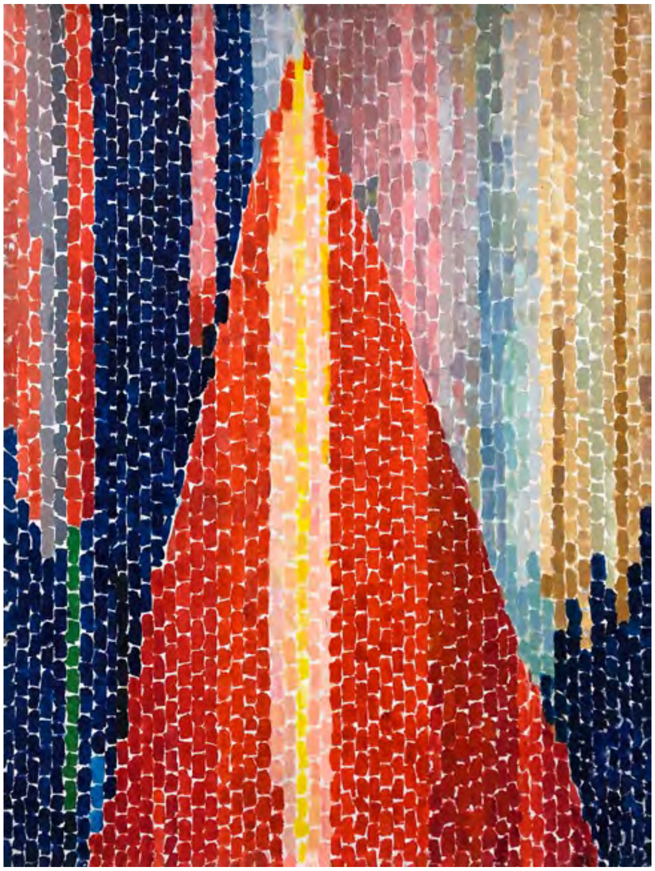
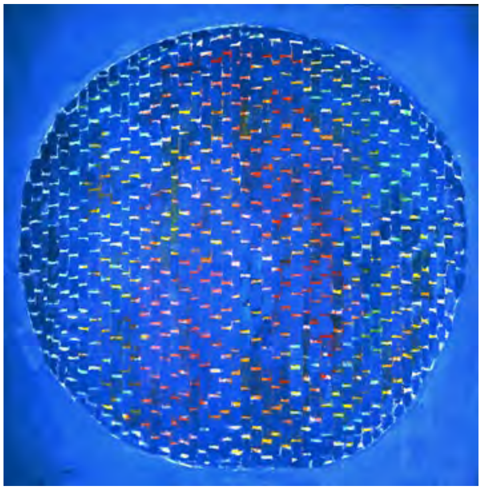
Laura Wheeler Waring, Portrait of a Lady (Alma Woodsey Thomas), 1947, oil on canvas, Smithsonian American Art Museum. Gift of Vincent Melzac, 1977. Public Domain.

Examples of Alma Woodsey Thomas' use of math to create tessellated art work



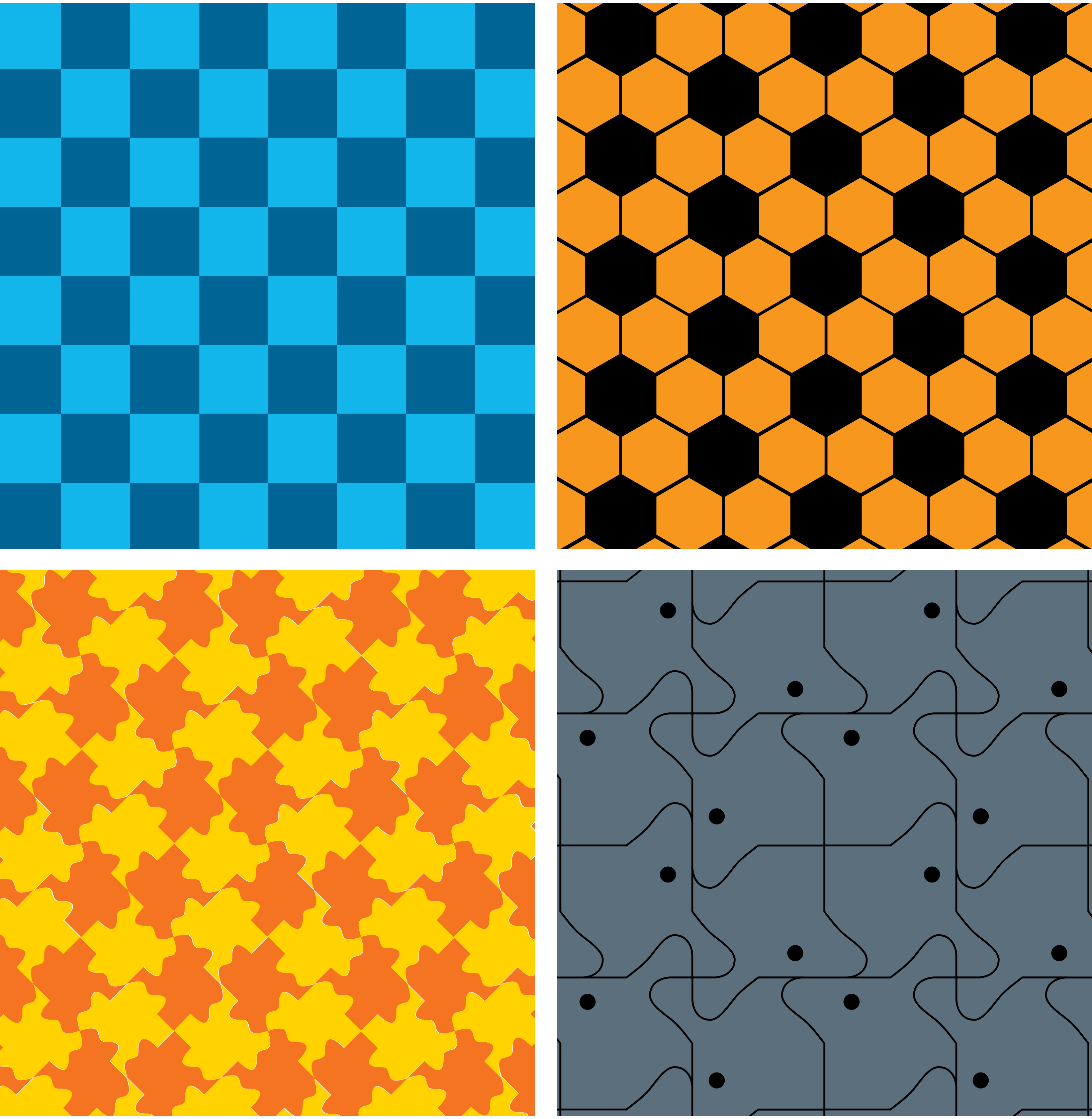
"Through color, I have sought to concentrate on beauty and happiness, rather than on man's inhumanity to man."

Alma Woodsey Thomas

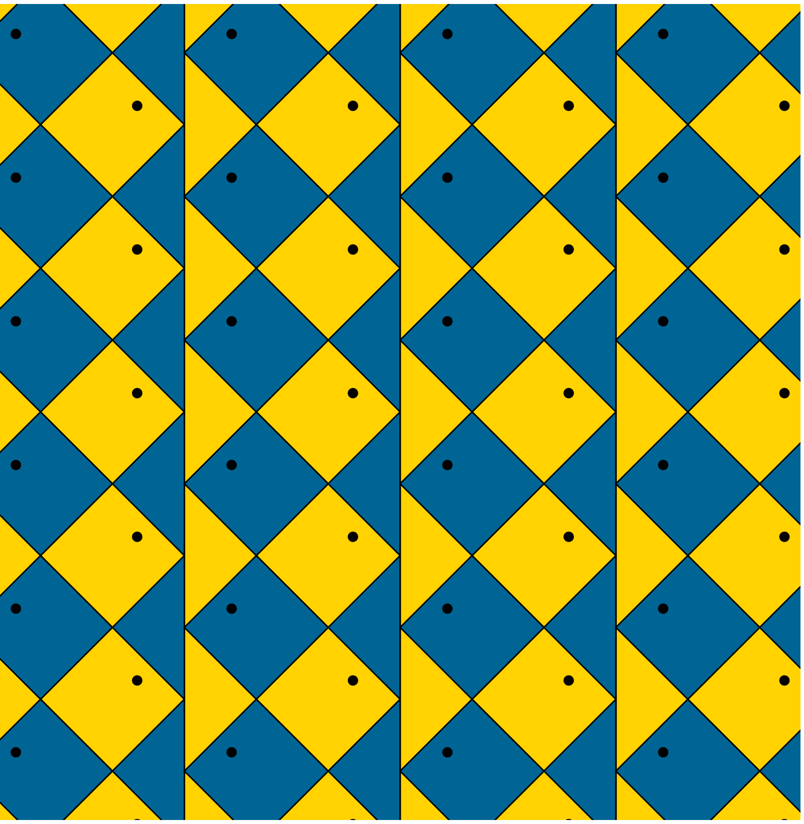


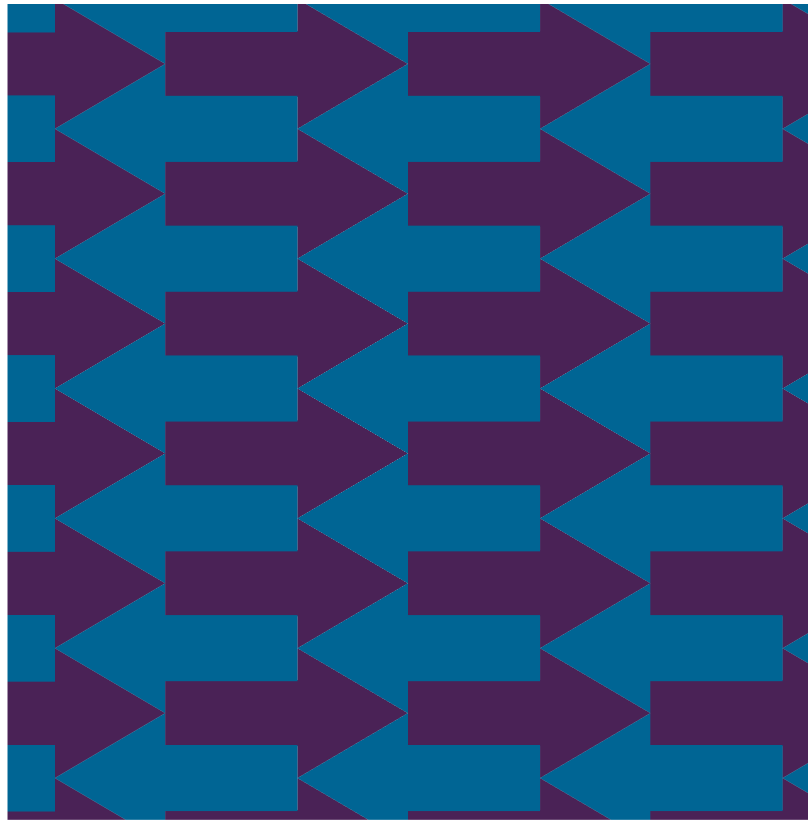
Inset above: Portrait of Alma Thomas © Michael Fisher, 1976.
Left: Alma W. Thomas, Blast Off, 1970, acrylic on canvas, 74 x 54 inches.
Right: Alma W. Thomas, Astronauts' Glimpse of the Earth, 1974, acrylic on canvas, 50.5 x 50.5 inches. Gift of Mr. and Mrs. Jacob Kainen.
These three images: Courtesy of the National Air and Space Museum. Fair Use, Educational Purposes.

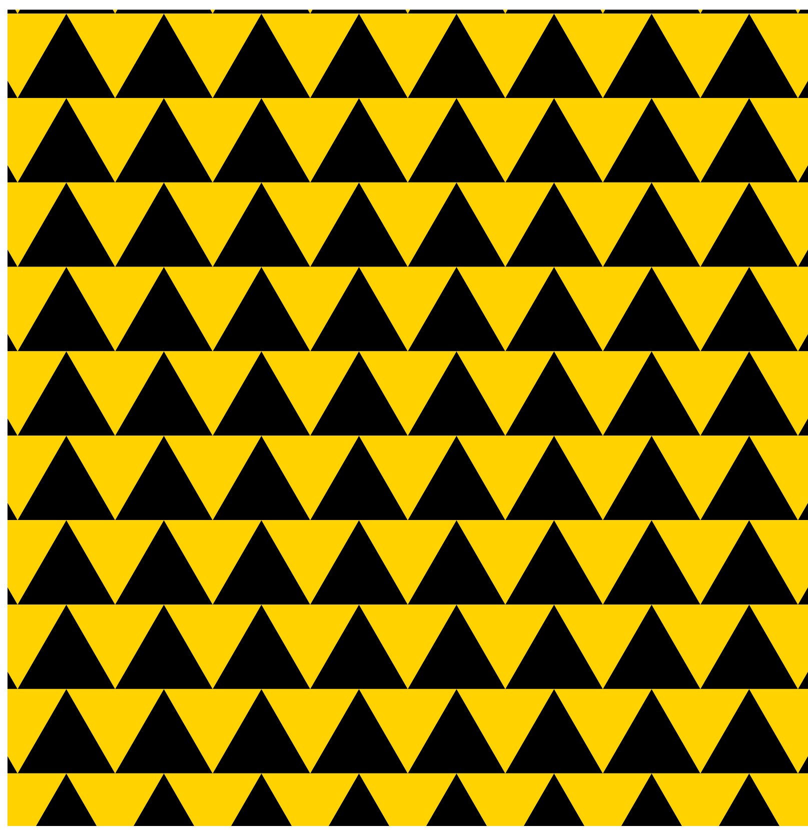
Examples of Regular Tessellations



Examples of Tessellated Patterns





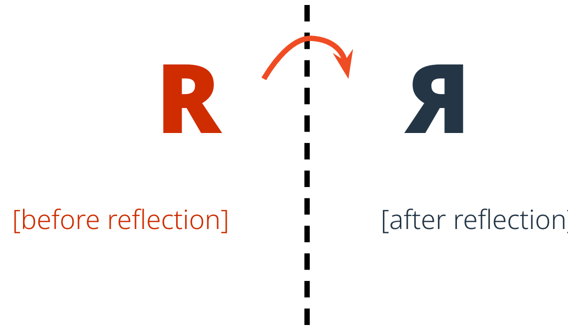


ASM+ HANDOUT

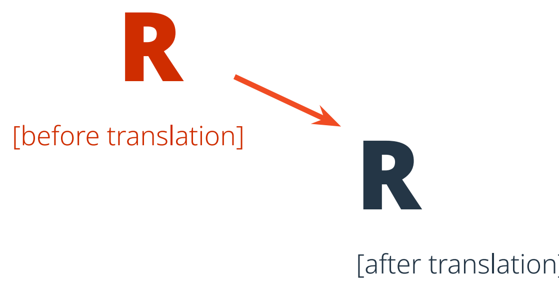
Transformations and Symmetry

Reflection
Repetition along an axis.

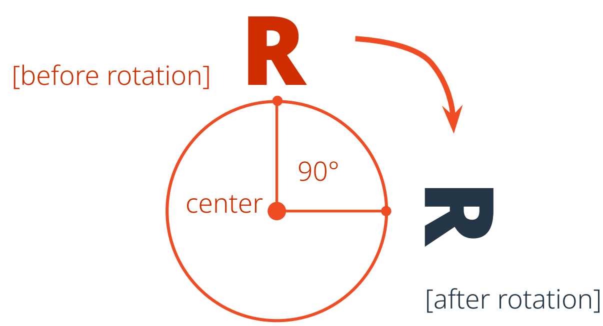
Canva



Translation
Repetition along a line.

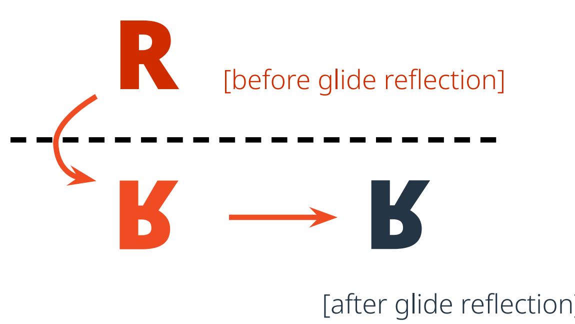


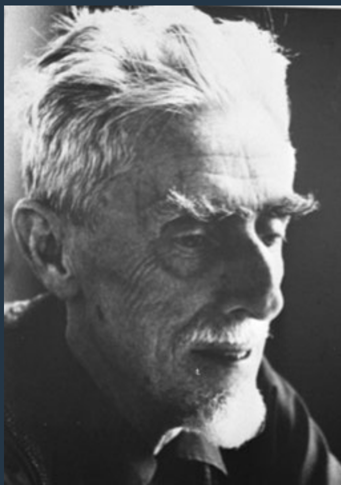
Rotation (Radial Symmetry)
Repetition along a point.



Glide Reflection (Translation + Reflection)
Reflected repetition along a line.

"M. C. Escher / Regelmatige Vlakverdeling," by Nina Stössinger on flickr, used under CC BY-SA 2.0 license, cropped.





"I believe that producing pictures, as I do, is almost solely a question of wanting so very much to do it well."

– M.C. Escher

Activity +2

ASM+
HANDOUT

M.C. Escher

Artist Biography

Maurits Cornelis (M.C.) Escher was born in Leeuwarden, Holland in 1898. He created a new school of art that used many mathematical ideas—especially geometry. Although he struggled in school, Escher liked to draw. When he was young his family wanted him to become an architect.

As a young man, Escher traveled to Spain and was fascinated by the tiled patterns created to decorate the Alhambra (an ancient Islamic palace in Grenada, Spain), and started creating tiled patterns in his drawings. He became inspired by mathematical ideas and the use of shapes in geometry.

Escher created art by taking geometric shapes and modifying the shapes so that they fit together like puzzle pieces to create very interesting patterns. His art became popular by the mid-1950s, and he became highly respected by mathematicians who identified the visualization of mathematical principles in his art. This was quite surprising since Escher had no formal training in math after high school!

He once said of his work, “For me it remains an open question whether [this work] pertains to the realm of mathematics or to that of art.”

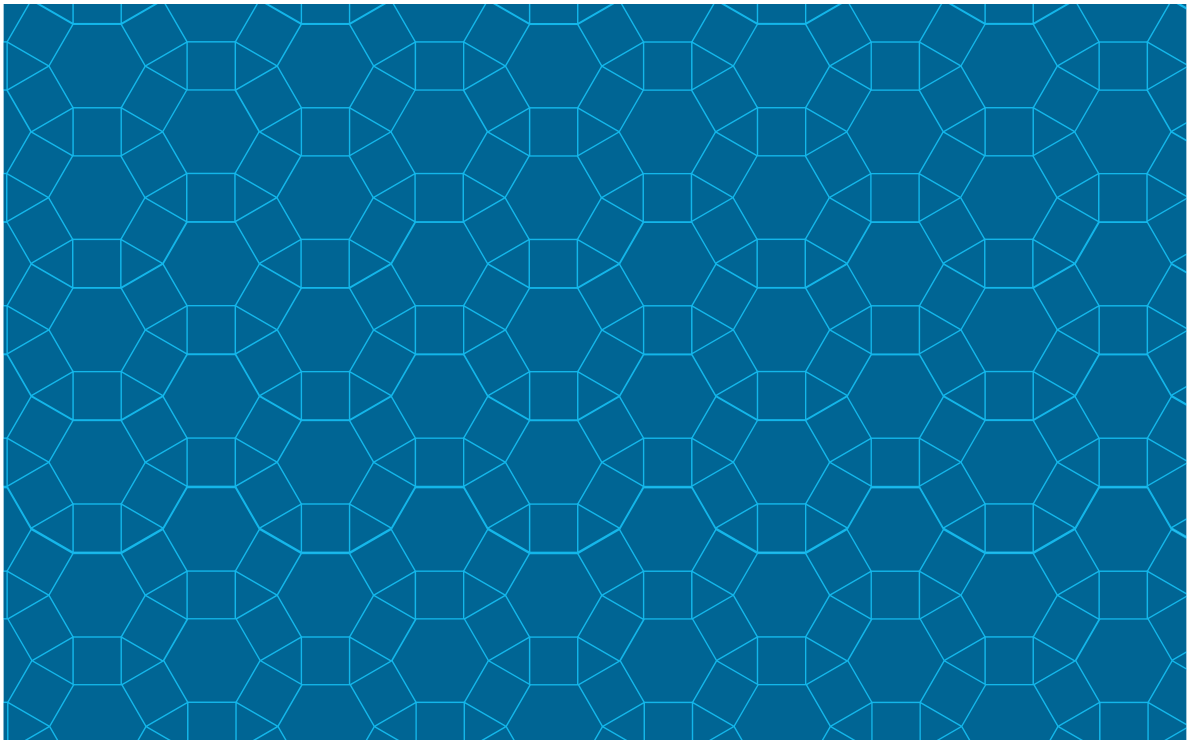
Pattern or Tessellation?

The object of this game is to work as a group to determine if the pictures show patterns and/or tessellations. Remember the rules of a tessellation! When your group has come to an agreement, write your responses on the lines below. Good luck!

1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________
5. ______________________________
6. ______________________________
7. ______________________________
8. ______________________________
9. ______________________________
10. ______________________________
11. ______________________________
12. ______________________________
13. ______________________________
14. ______________________________
15. ______________________________
16. ______________________________

Pattern or Tessellation Game Samples

1.



FHI 360 staff

2.

FHI 360 staff

Activity +3

Pattern or Tessellation Game Samples

3.

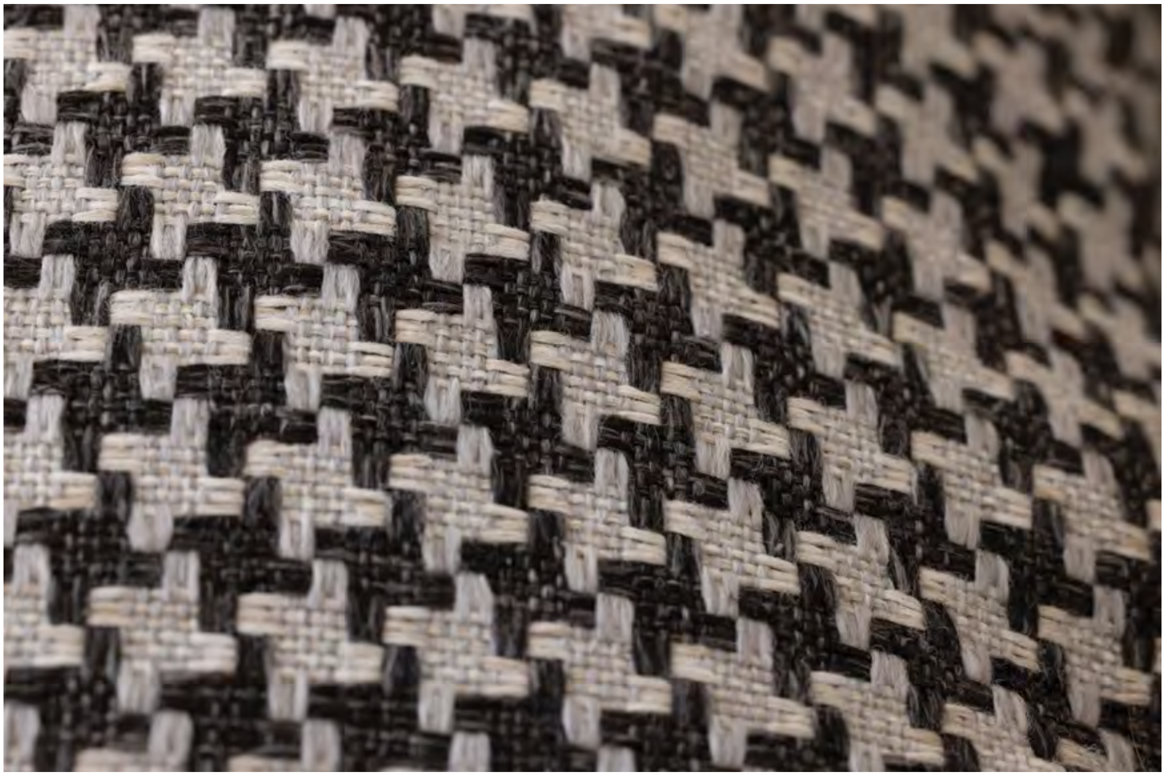


Photo by https://unsplash.com/@enginakyurt on [Unsplash](https://unsplash.com)

4.

FHI 360 staff

Activity +3

Pattern or Tessellation Game Samples

ASM+
HANDOUT

5.

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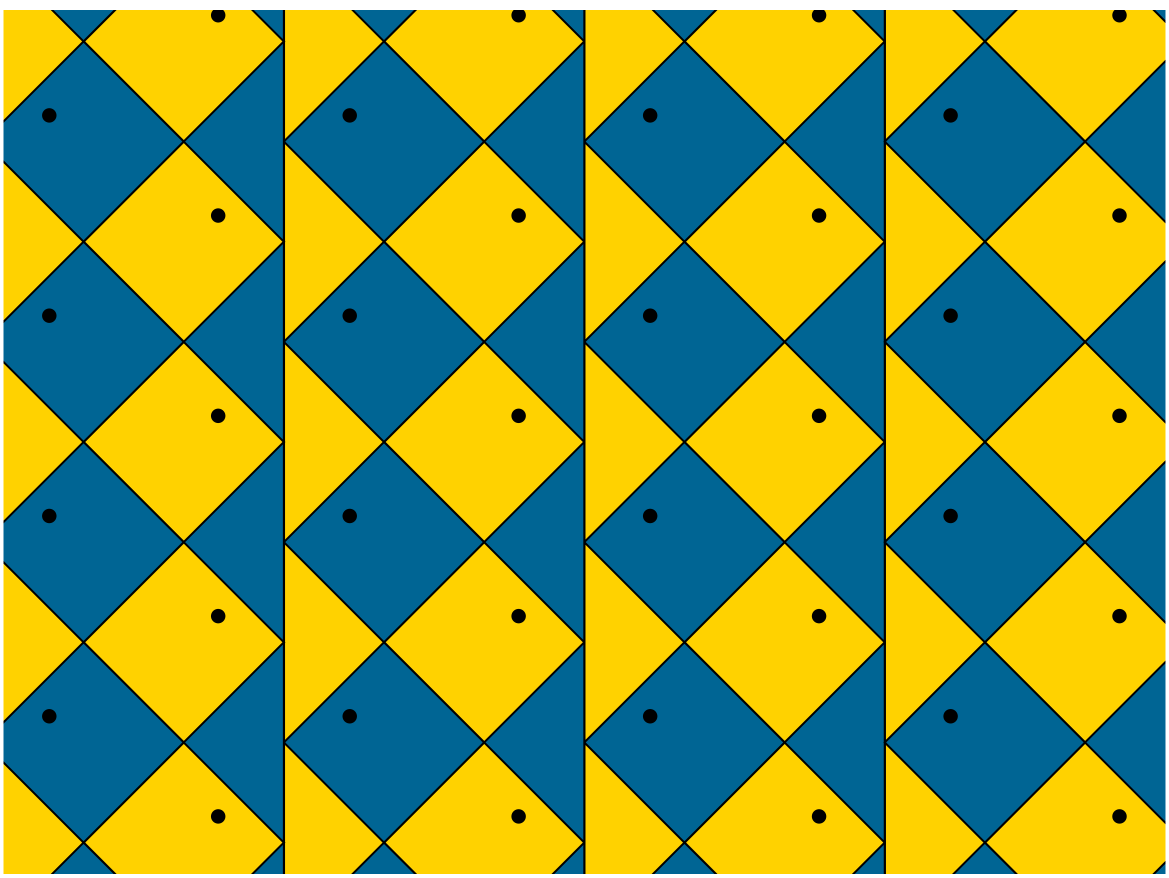

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Pattern or Tessellation Game Samples

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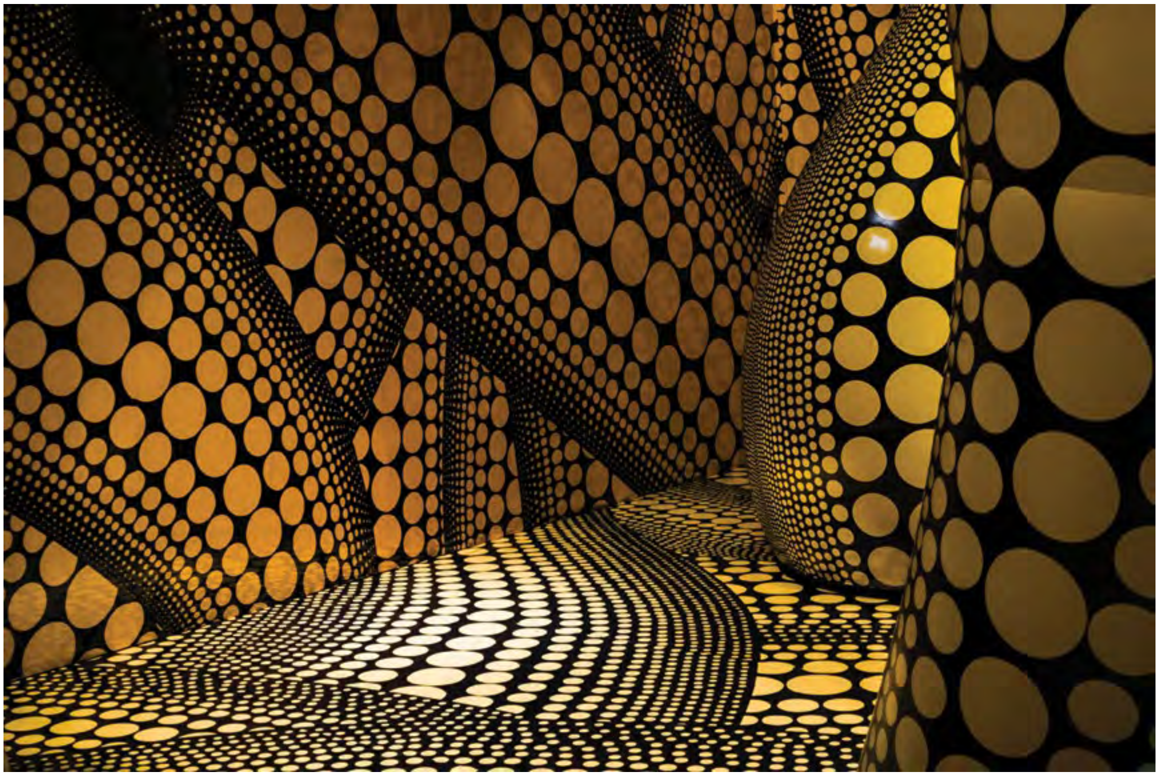

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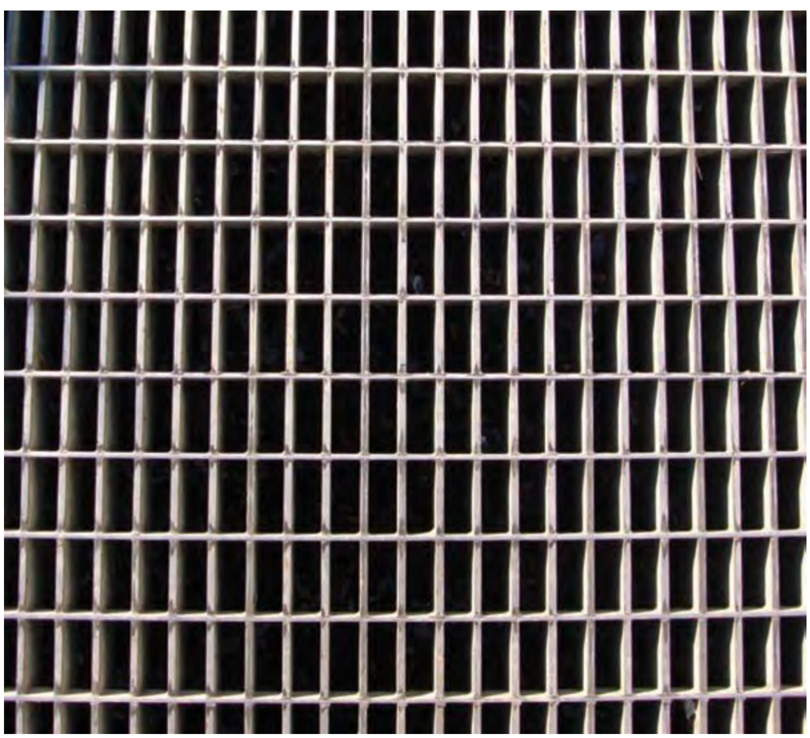

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Pattern or Tessellation Game Samples

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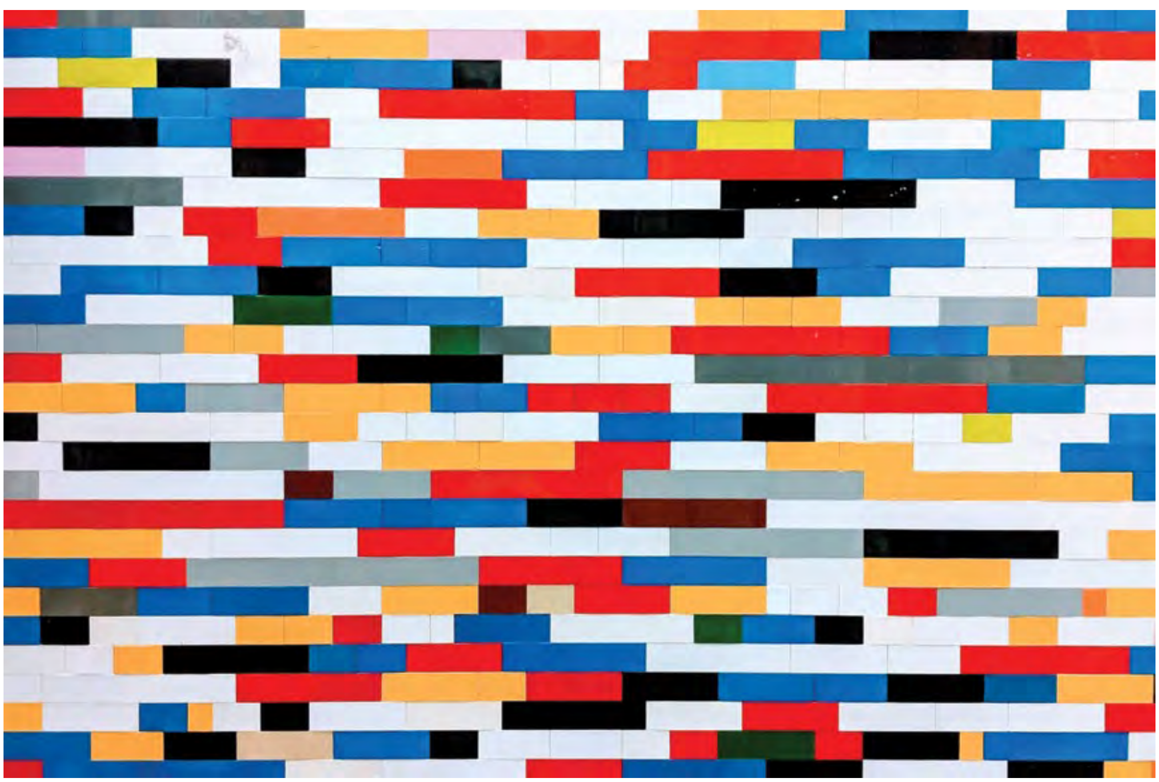


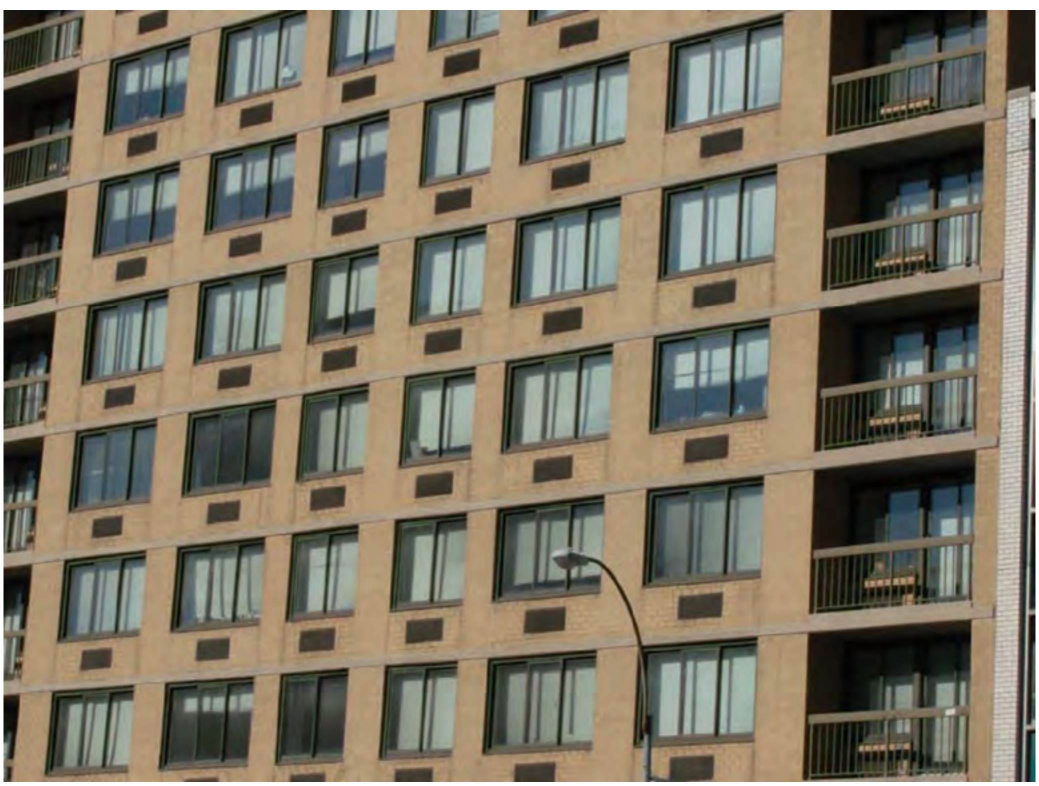
Photo by https://unsplash.com/@designedbyflores on Unsplash

Pattern or Tessellation Game Samples

11.

Canva

12.



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Pattern or Tessellation Game Samples

13.

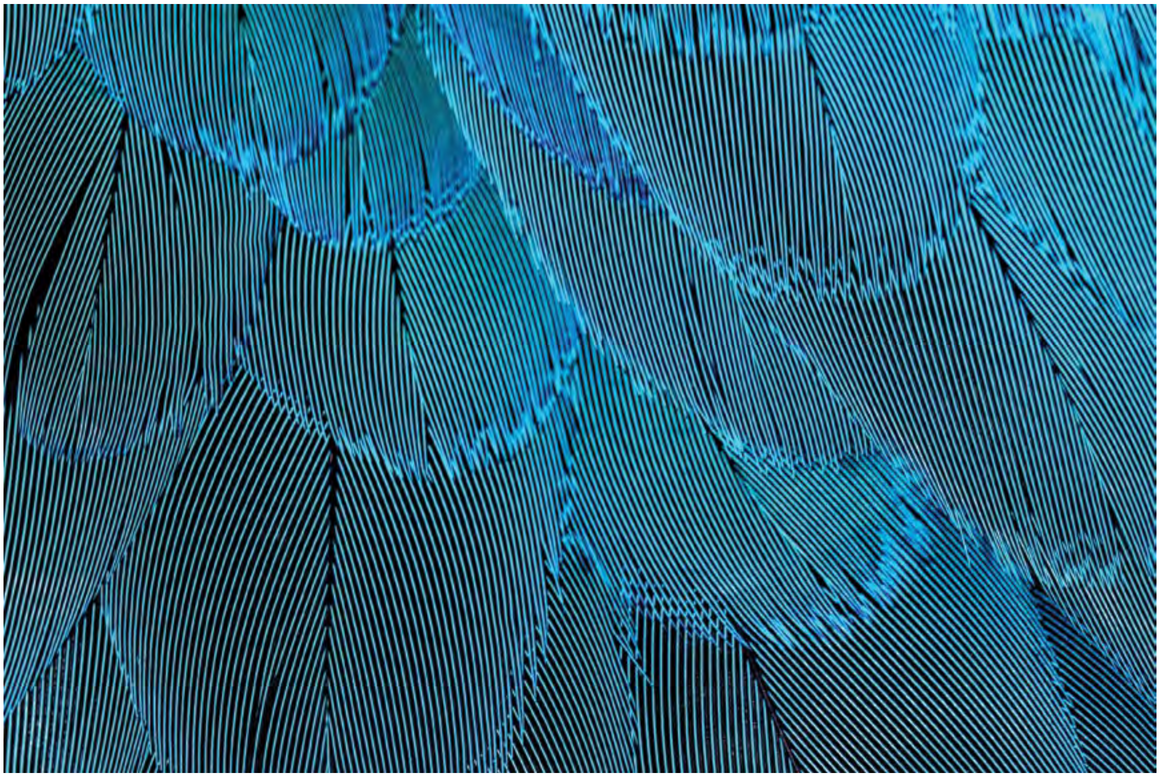

Photo by https://unsplash.com/@davidclode on Unsplash

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Photo by https://unsplash.com/@steve_huntington on Unsplash

Pattern or Tessellation Game Samples

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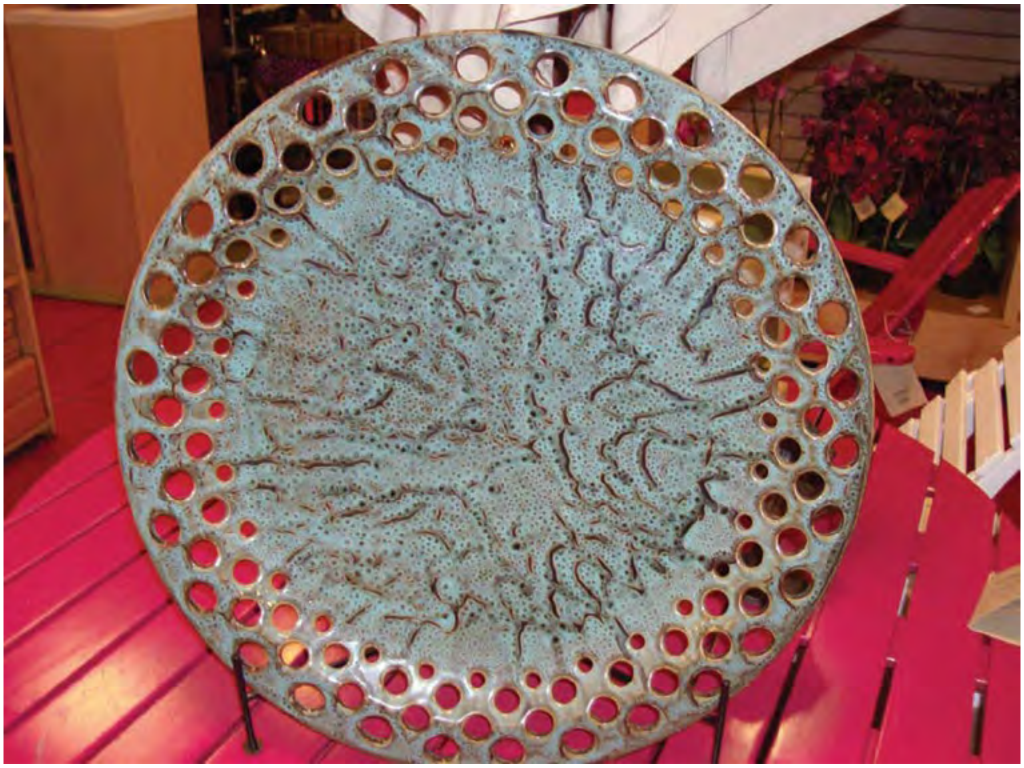

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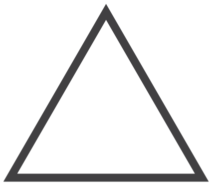
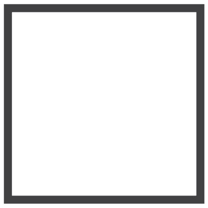
16.

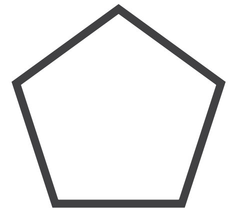
Canva

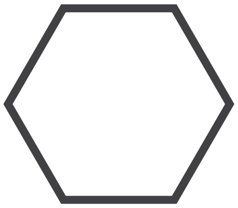
Geometric Shapes


Triangle


Square


Rectangle


Pentagon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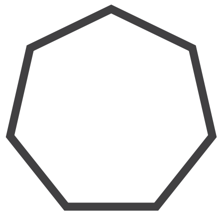
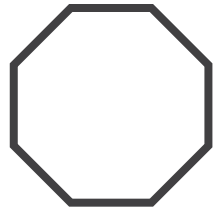
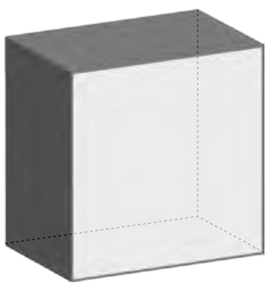
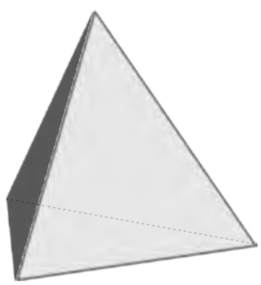
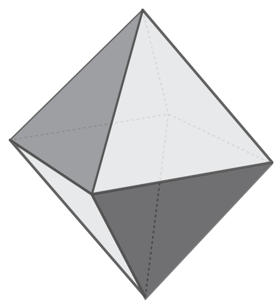
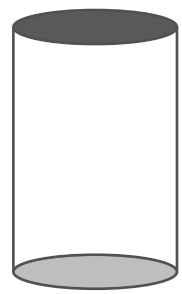

Heptagon

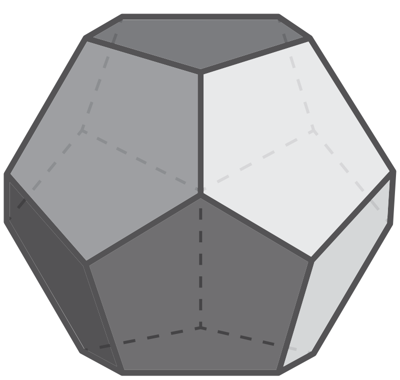

Octagon

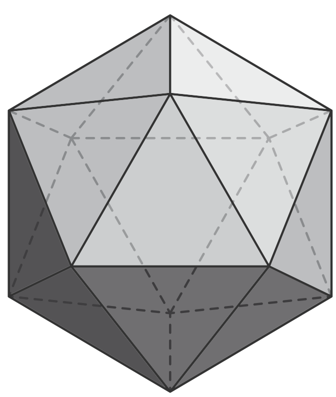

Cube

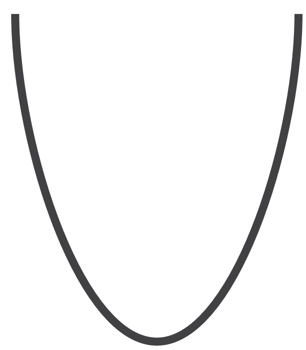

Tetrahedron


Octahedron


Cylinder


Dodecahedron


Icosahedron


Parabola

Activity +4

Piet Mondrian

Artist Biography

ASM+ HANDOUT

"Art is not made for anybody and is, at the same time, for everybody."

– Piet Mondrian

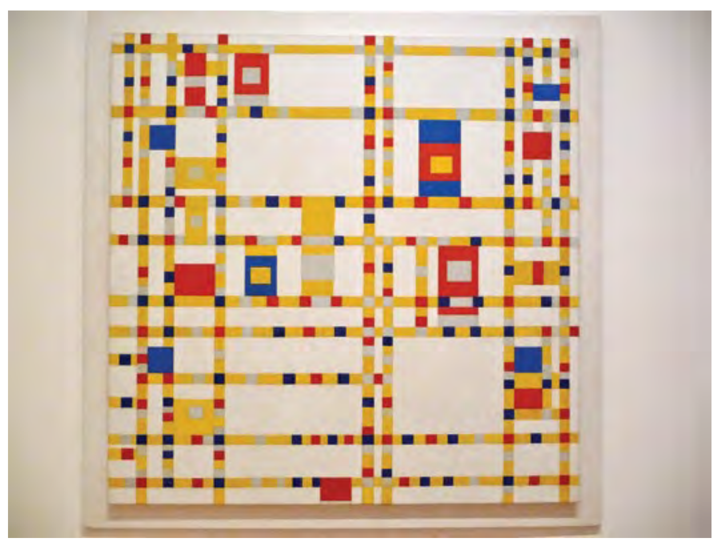
"**Piet Mondrian – Self-Portrait**," by **Kent Wang** on flickr, used under **CC BY-SA 2.0** license, no changes made.

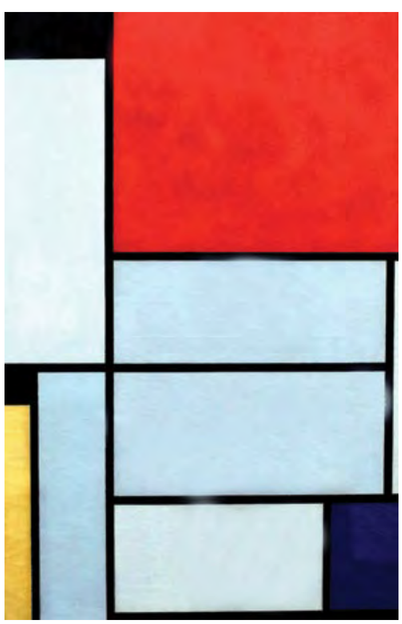
Piet Mondrian was born in 1872 in Amersfoort, the Netherlands. His first name was Pieter, but he shortened it to Piet. Mondrian always knew he wanted to be an artist. He studied at the Rijksakademie van Beeldende Kunsten in Amsterdam. Until 1908, his work was naturalistic and traditional. His paintings were mostly still-lifes and landscapes.

Mondrian studied the works of artists like George Braque and Pablo Picasso, who were experimenting with other styles and started painting in less traditional ways. When World War I broke out, Mondrian was in the Netherlands. It was there that he refined his style, by using fewer colors and geometric shapes. This new style was called Neoplasticism, a school of art using formal structures and primary colors.

During World War II, Mondrian moved to New York City. There he met many American abstract artists and published essays on Neoplasticism. His work started to reflect life in New York City and in 1942, his first solo show took place in there. One of his most famous works is *Broadway Boogie Woogie*, now in the collection of The Museum of Modern Art in New York City.

It is important to note that, while much of Mondrian's later art contains geometrical shapes, this does not mean that Mondrian directly used mathematics in his approach. In fact, a 1995 exhibition demonstrated that the geometry in Mondrian's paintings were arrived at through trial and error until a balance of lines and planes was right.





Clockwise from top left: Piet Mondrian, Composition II in Red, Blue, and Yellow, 1930, Public domain, via Wikimedia Commons. "Broadway Boogie Woogie," by Martin Dougiamas on flickr, used under CC BY 2.0 license, no changes made. "IMG_5899. Piet Mondrian. 1872-1944; Paris. Tableau 1. 1921. Cologne. Musée Ludwig.," by jean louis mazieres on flickr, used under CC BY-NC-SA 2.0 license, no changes made. "Piet Mondrian," by rocor on flickr, used under CC BY-NC 2.0 license, no changes made. Piet Mondrian, Tableau I, 1921, Public domain, via Wikimedia Commons.

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Field Trip Viewing Sheet

Name:

Location:

Title of Art: *Artist:*

Are there geometric shapes in the art? Which ones?

Can you identify any patterns? Can you describe them?

Are there tessellations in the art? If so, what kinds of symmetry transformations do you see (translation, reflection, rotation, glide reflection)?

How does the artist use asymmetry?

What colors does the artist use? Why might the artist have chosen to use colors this way?

What words would you use to describe the art?

In the space below, sketch the art, something the art reminds you of, or something the art makes you feel.

Gordon Sasaki

Artist and Educator Biography

"Gordon Sasaki Headshot," The National Endowment for the Arts. Fair Use, Educational Purposes: **https://www.arts.gov/stories/podcast/gordon-sasaki**

Art has been an integral part of Gordon Sasaki's life since childhood. In grade school, he always found the arts to be an immediate and important means of expression and communication.

In third grade, Sasaki's teacher selected him to illustrate the image of Santa Claus on the school's Christmas mural. He felt a bit shy about the notoriety, but after completion he was proud and happy to share his abilities with his fellow classmates. This seemingly insignificant occurrence boosted Sasaki's sense of confidence and self-esteem.

In fifth grade, Sasaki had a teacher who was an artist himself and allowed his class extra art sessions during the week. He talked about and showed the students examples of different types of art and gave them many invaluable tips on techniques in drawing and painting. This teacher was a significant influence on Gordon Sasaki's decision to become an artist.

While studying art in college in 1982, Sasaki had a serious automobile accident that injured his spinal cord and resulted in the use of a wheelchair for basic mobility. While his life changed overnight, he still had his art.

Sasaki returned to college to graduate with high honors and earn a graduate degree in fine arts. Since then, art has enabled him to travel the world, sharing and experiencing many new things and different points of view. For several years, Sasaki lived in New York City and worked in the education department of The Museum of Modern Art (MoMA), where he was an artist in residence and educator. He is still a practicing artist, currently living in his hometown of Honolulu.

"The reason why I started was because I didn't see anything that was talking about disability in a positive light. ... I was really thinking about can a painting of a wheelchair be beautiful. Can disability be seen in a different frame? So, I started these works."

– Gordon Sasaki

At MoMA, Gordon Sasaki conducted classes and tours for students of all ages on many topics, including how art and mathematics are connected and how incorporating mathematical principles (like the golden ratio and tessellations) into artwork makes the art more aesthetically pleasing.

In his photo above, Gordon Sasaki sits in front of some of his artwork: *Gold Wheelchair*, a bright red wheelchair painted over gold leaf; *Black and White Wheelchair*, and *Silver Wheelchair*.

Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

+8 Resources

Books

The Art of Shapes: For Children and Adults
by Margaret Steele and Cindy Estes (Los Angeles: Museum of Contemporary Art, 1997). This book uses figures extracted from contemporary works of art to present various shapes, including the star, diamond, and cone.

Color Your Own Modern Art Masterpieces
by Muncie Hendler (New York: Dover Books, 1996). Color 29 meticulously rendered drawings of great 20th-century abstract art.

The Magic Mirror of M.C. Escher
by TASCHEN (Cologne: TASCHEN GmbH, 2022). This updated and redesigned edition provides biographical data, 250 illustrations, and a breaking-down of Escher's use of math in his art.

Math Art and Drawing Games for Kids: 40+ Fun Art Projects to Build Amazing Math Skills
by Karyn Tripp (Quarry Books, Illustrated edition, 2019). Create fine art-inspired math projects, including M.C. Escher's tessellations, Wassily Kandinski's abstractions, and Alexander Calder's mobiles. Learn about patterns and motifs used by cultures from all over the world, including Native American porcupine quill art, African Kente prints, and labyrinths from ancient Crete.

Math Art: Truth, Beauty, and Equations
by Stephen Ornes (Union Square & Co., 2019). Includes examples of art that explores more complex math.

Math Line Designs From Around the World Grades 4-6: Dozens of Engaging Practice Pages That Build Skills in Multiplication, Division, Fractions, Decimals, and More
by Cindi Mitchell (Scholastic Teaching Resources, 2nd edition, 2008). Dozens of activity pages featuring striking designs based on motifs from cultures around the world. Students solve math problems and then follow a key to color the designs.

Math Skills Made Fun: Kaleidoscope Math: Dozens of Reproducible Activities That Will Dazzle Kids and Build Skills in Addition, Subtraction, Multiplication, Division, and More
by Cindi Mitchell (New York: Instructor Books, 2003). Eye-popping, color-in designs let young people explore symmetry and build math skills.

Math-Terpieces: The Art of Problem-Solving
by Greg Tang (New York: Scholastic Books, 2003). Challenge students with this innovative approach to math and the use of art history to expand creative problem-solving.

M.C. Escher Kaleidocycles
by Doris Schattschneider, Wallace Walker, and TASCHEN (Cologne: TASCHEN GmbH, 2022). This fun activity book for students includes 17 easy-to-assemble paper sculptures transforming M.C. Escher's geometric designs into three-dimensional polyhedra.

M.C. Escher: Symmetry Coloring Book
by Pomegranate (PomegranateKids, 5th revised edition, 2010). Features 22 of M.C. Escher's symmetry drawings for students to color.

M.C. Escher: The Graphic Work
by TASCHEN (Cologne: TASCHEN GmbH, 2016). Features a biography, chronology of Escher's work, and approximately 100 illustrations with explanatory captions.

M.C. Escher: Visions of Symmetry
by Doris Schattschneider (New York: W.H. Freeman & Co., 1992). A view of notebooks, periodic drawings, and related work of M.C. Escher.

Modern Art for Kids: Hands-On Art and Craft Activities Inspired by the Masters (Art Stars series)
by Stephanie Ho Poon and Shannon Yeung (Quarry Books, 2023). An illustrated hands-on guide with fun facts, artist stories, and arts and crafts activities to inspire curious and creative kids with interesting art topics.

Tessellation Patterns Coloring Book
by John Wik (Dover Publications, Creative Haven edition, 2013). A collection of 30 designs to color. *Note: The Dover Creative Haven line features several tessellation coloring books.*

Websites

Alma Woodsey Thomas (National Museum of Women in the Arts)
This page provides a brief biography of Thomas and links to the works and exhibitions available through the NMWA museum (Washington, D.C.). *Note: This site may be useful if your group is doing a virtual field trip.*

Art Beyond Sight
ABS is dedicated to empowering and enriching the lives of thousands of children and adults through the life-enriching benefits of art and culture, bringing access, inclusion, and promising opportunities to people with all types of disabilities on a local, national, and global scale.

The Art of Alma W. Thomas: A Colorful Response, The Smithsonian National Air and Space Museum
This article by curator Carolyn Russo explains the influence of the Apollo moon landings and nature on Thomas's paintings. *Note: This site may be useful if your group is doing a virtual field trip.*

Challenging Notions: Accessbility and the Arts, *American Artscape*, 2015 No. 1
This issue of *American Artscape*, the magazine produced by the National Endowment for the Arts, celebrates the 25th anniversary of the passing of the Americans with Disabilities Act (1990) by highlighting organizations and individuals that make art accessible to every American. The issue is free to download as a pdf on arts.gov.

The History of the Kaleidoscope and David Brewster
A brief explanation of Brewster's life and the invention and improvements to the kaleidoscope.

Mathematics and the Art of M.C. Escher
This podcast episode from the National Gallery of Art includes a lecture Doris Schattschneider gave at the National Council of Teachers of Mathematics annual meeting in 2018. She discusses how the imagery in Escher's graphic works makes use of geometry and provides visual metaphors for abstract mathematical concepts. *Note: Playing a clip from this lecture may be useful when discussing Schattschneider's biography.*

Mathematics Made Visible: The Extraordinary Mathematical Art of M.C. Escher
This short clip from BBC Four provides a brief introduction to M.C. Escher's life and work and discusses his work with tessellations.

Mathigon – The Mathematical Playground
This site includes free tools, courses, and manipulatives for more interactive and engaging online math learning. The Polypad allows students to view and manipulate examples of several different kinds of shapes and patterns and to create their own. The Courses, Activities, and Lessons sections provide more information to support math learning, including both tessellations and Mondrian's rectangles tasks. There is also an e-module on symmetry that may be helpful for these activities.

M.C. Escher Collection
The official website of the M.C. Escher Foundation and The M.C. Escher Company, including current and upcoming exhibitions of Escher's work, an online gallery with most of his works, and a more detailed biography.

Mondrian Art Math Activity
These handouts from Math Circle Network can help students understand how Mondrian's rectangles reflect math principles.

Piet Mondrian (Guggenheim)
This page provides a brief biography and compiles links to works by Mondrian in the collection at the Solomon R. Guggenheim Museum (NY). *Note: This site may be useful if your group is doing a virtual field trip.*

Piet Mondrian (MoMA)
This site compiles all the works, exhibitions, audio, and publications related to Piet Mondrian on the MoMA (Museum of Modern Art, NY) site. *Note: This site may be useful if your group is doing a virtual field trip.*

Symmetry Challenge
One of the games available on Coolmath Games, the Symmetry Challenge teaches students to understand symmetry by requiring the user to complete the image.

Tessellation Creator
This online tool from the National Council of Teachers of Mathematics allows students to create and manipulate tessellations using different geometric shapes.

What inclusive, accessible arts education looks like (Perkins School for the Blind)
This article provides four tips for making art and music more accessible.

Afterschool Math Plus

Revised Edition

Theme +1: ArtMath

+9 Glossary and Materials List

Glossary

Abstract art In the twentieth century some painters and sculptors broke away from the traditional representation of objects. Abstract art uses shapes and colors to represent objects—traditional art represents objects by using recognizable images.

Algebra The mathematics of variables.

Angle The distance measured in degrees between two lines that meet.

Asymmetry A shape that cannot be divided into two parts that are mirror images of each other. In other words, asymmetrical means "not symmetrical."

Circumference Distance around a circle or sphere.

Congruent The same in size and shape.

Diameter A line segment between two points on a circle or sphere that passes through the center.

Fibonacci sequence A series of numbers named after Leonardo of Pisa, known as Fibonacci. Every number is the sum of the previous two numbers starting with the numbers (0,1,1). The series continues on to infinity 0,1,1,2, 3, 5, 8, 13, 21, 34, 55, 89, etc. As the numbers in this series get larger, the ratio of one number to the number before it will get closer and closer to the golden ratio (1:1.61803) (see below). This sequence of numbers describes many aspects of nature. For example, the way the florets of a sunflower are arranged. It is also found in art, including the work of Mondrian and DaVinci.

Golden ratio The golden ratio is an irrational number with the approximate value of 1.6 (1.618033988749...). Many architects and artists, for example, Leonardo da Vinci, use the golden ratio in their work, either intentionally or instictively, because of its supposed pleasing proportions. The golden ratio also appears in nature; an example is the ratio of the length of a human hand compared with its width. The golden ratio is known mathematically as phi after the Greek sculptor Phidias who studied and used the ratio extensively.

Golden rectangle A rectangle where the ratio of its length and width equals (approximately) 1.6, the golden ratio. If a square is removed from a golden rectangle, another golden rectangle with the same proportions remains.

Measurement The process of assigning a number to a physical property like distance or volume.

Neoplasticism A school of art that uses a very formal structure for a work of art. The artists use vertical and horizontal images (lines) and restrict their colors to black, white, and the primary colors.

Pattern A repetitive set of objects or numbers.

Plane A flat, two-dimensional surface.

Plane figure A geometrical configuration all of whose points lie in a plane.

Polygon A plane figure with at least three straight sides and angles, and typically five or more.

Prediction A guess on an outcome.

Prescribed angle A specific angle.

Radius Half a diameter, or any line segment from the center to any point on a circle or sphere.

Reflection A transformation that "flips" a figure over a mirror or reflection line. A reflected pattern has perfect symmetry.

Regular polygon A geometric shape where all sides are equal lengths to each other.

Rotation A transformation that turns a figure about a fixed point at a given angle and a given direction.

Symmetry An exact correspondence of form or shape on opposite sides of a dividing line, a plane, or about a center or axis.

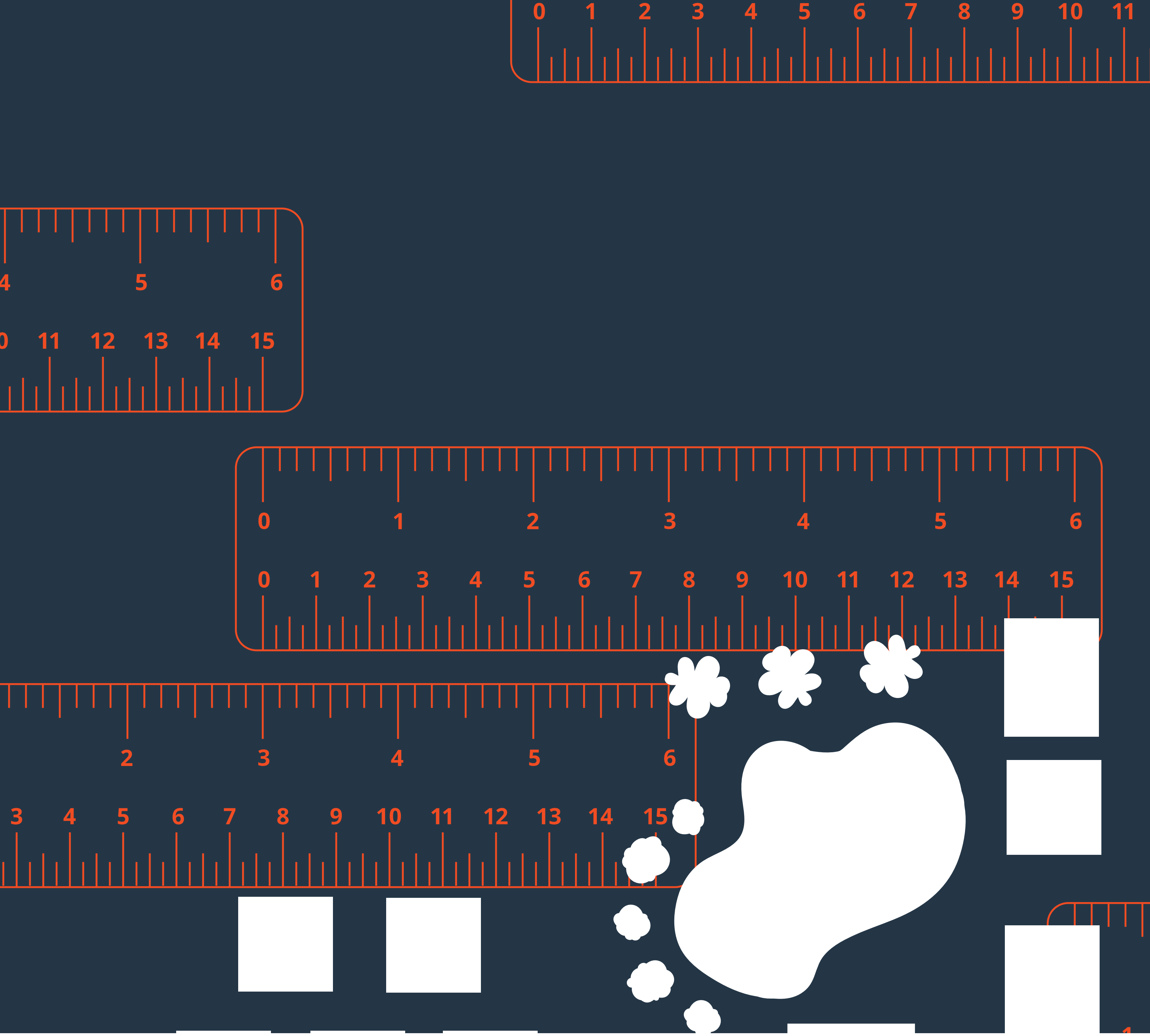
Tessellation A repeating pattern of figures that covers a plane without any gaps or overlaps.

Transformations Operations that change the appearance or location of a plane figure. The four common transformations are:

- translation—repetition along a line
- reflection—repetition along an axis
- rotation—repetition around a point
- glide reflection—a combination of translation and reflection.

Materials List

- ☐ Beads—clear and color transparent
- ☐ Can opener
- ☐ cardboard
- ☐ Clear tape
- ☐ Compass
- ☐ construction paper
- ☐ Crayons
- ☐ Glitter
- ☐ Glue
- ☐ Glue stick
- ☐ Markers
- ☐ Notepaper
- ☐ Paints
- ☐ paint brushes
- ☐ Paper
- ☐ Pencils
- ☐ pens
- ☐ Permanent markers
- ☐ Pipe cleaners
- ☐ Protractors
- ☐ Rulers
- ☐ Sand
- ☐ Sandpaper
- ☐ Scissors
- ☐ Stackable chip can (like Pringles), with the plastic cap
- ☐ Sticky notes (square, various colors)
- ☐ Transparency film
- ☐ water
- ☐ Wax-covered yarn



Afterschool Math Plus

Theme +2: Built Environment

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Canva

Introduction

You can't get any more real world than the built environment that surrounds our everyday lives. Geometrical shapes—cubes or spheres, ovals or parabolas, triangles and rectangles—are what make up the buildings where we live, play, and work. In this series of activities, students explore their environment, looking at windows, doors, buildings, streets, and fences with a mathematical eye.

Students make their own metric measuring tapes and construct a tool out of rope to measure, collect data, convert it to scale on graph paper, and draw representational maps. Students use what they have learned about scale, measurement, and their environment to create a blueprint for an ideal community in a design session called a charrette.

In thinking about the community they want to create, students discuss issues of accessibility, safety, and social justice (e.g., Are health facilities available for everyone? Are places for older people well designed?) At the museum, students use their blueprints as a guide to build a scale model of their ideal community.

Equity

Designing an ideal community requires students to consider the needs of all community members—children, families, older adults, people who are or are not living with disabilities—with respect and concern. Each student brings a different, and valuable, perspective to the task. The theme allows students of color and students who identify or present as girls, groups who are traditionally underrepresented in math and science careers, opportunities to experience the role of an architect alongside students of groups who are more represented in the field.

NCTM Math Standards

Content Standards

- **Data Analysis and Probability:** Collect data using observations; represent data accurately.
- **Measurement:** Understand attributes such as length and width and select appropriate unit of measurement; understand need for standard units; use customary and metric units; solve problems using scale; and explore ratio.
- **Algebra:** Understand patterns, relations, and functions.
- **Geometry:** Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- **Number and Operations:** Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Process Standards

- **Problem Solving:** Solve problems as they arise; and apply and adapt a variety of appropriate strategies to solve a problem.
- **Reasoning and Proof:** Select and use various types of reasoning.
- **Communication:** Organize and consolidate mathematical thinking through communication.
- **Representation:** Create and use representation to organize, record, and communicate ideas.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

Inclusion

The Built Environment theme has strategies throughout to ensure the participation of students with a broad range of disabilities, many of whom will be able to draw on their first-hand experience with issues around access during these activities. Their expertise and problem-solving skills will enhance the design of the ideal community. Are building entrances and sidewalks ramped? Are signs in elevators or lobbies in Braille?

Some tips for inclusion:

- Maps, drawings, and measuring tools can be made tactile for students with visual impairments. Having a supply of wax-covered yarn, sand, pipe cleaners, and glue handy will facilitate making charts and maps tactile.
- One way to include a student with limited fine motor skills is to use a "hand over hand" technique or to allow the student to provide directions to another person who will draw the map.
- Ask students to work in pairs as they measure structures. Students can share tasks to complete them. This will encourage teamwork and communication.
- Give students the opportunity to problem-solve their situation. Many students with disabilities have already figured out their own way of accomplishing things.
- Encourage students with and without disabilities to participate equally. Each will bring their own strength to the program.

Cultural Links

The Built Environment theme provides a wealth of connections to students' own culture and family history. Looking at pictures of houses and homes from around the world online or in books and magazines can provide an interesting starting point for the theme, e.g., stone houses in Italy, round homes in Kenya, Navajo hogans in the United States, houseboats in the Netherlands, stilt houses in China, and bamboo homes in Bali.

Students' ideal community may reflect what is happening in the world. For example, at one site, because students were creating their ideal community following a tsunami, they decided to build a tsunami-proof community. The result was a community that had wave-breakers in the water, municipal buildings set far back from the shoreline, and a temporary orphanage where children could wait to be rejoined with their families.

Literacy and Math Identity

The architecture-based math identity activities in this theme help all students develop essential math skills for future careers. The role models—Karen Braitmayer, Daniel Glenn, Zaha Hadid, Francis Kéré, and Maya Lin—provide a look at contemporary architects working on significant structures. Be sure to read their bios and have students try the related career and role-model activities.

After a field trip to the New York Hall of Science in Queens, New York, students at the Chinese American Planning Council afterschool program were given the hypothetical challenge of building a site for the Summer Olympics in Flushing Meadow Park. The students researched the measurement of the park area online and fit their structures into the existing space. They built scale models of the Unisphere, Citi Field, the New York Hall of Science, and the waterways that surround the park. The students also researched the law about parking spots for people with disabilities and included the correct percentage in their models. They labeled bathrooms in English and Chinese and included a dragon boat racecourse in the Olympic park. In Huntsville, Alabama students from the University Place School afterschool program who partnered with the U.S. Space and Rocket Center, built a Mars colony with recyclable materials from the Space Camp. In Bangor, Maine, students from the Indian Island After-School program working with the Maine Discovery Center built an ideal community on the Penobscot river that reflected the values of the Wabanaki culture.

Remember to:

- ☐ **Check out the resources.**
- ☐ **Review the glossary.**
- ☐ **Send out the family letter when you start the theme.**

At the Museum

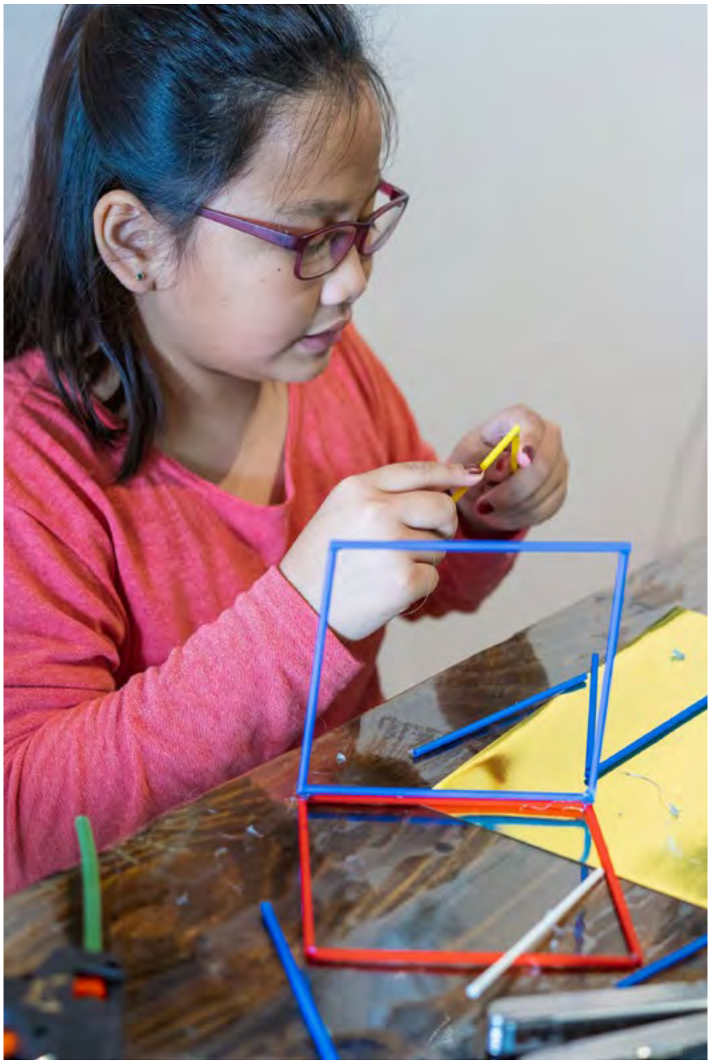
During their sessions at the museum, students use a blueprint they have created in a charrette as a guide for building a scale model representing their ideal community. In architecture, interior design, landscape design, graphic design, etc., a charrette is generally a collaborative design session that incorporates the ideas and creativity of several people, especially people living in the affected neighborhood or local area who will be using or affected by the space. The term originally meant an architectural student's final work representing what they had learned.

The student-built scale models can be made from found objects such as cartons, paper tubes, cereal boxes, and other recyclables. Or, they can be built out of foam board cut to scale.

A fun idea for the culminating event is to have students set up a mock press event at which they explain why they added certain features to their ideal environment. After the press conference, students can walk guests around the model, explaining the details of the community they created. It is important for all students to have a role in the culminating event.

The museum itself is an interesting built environment that is devoted to science and, like all science spaces, is also full of math. During an orientation trip, students can be helped to find the math within the exhibits.

Students also can observe how the exhibit space is organized. They will discover that, in addition to the hands-on displays, there are classrooms, libraries, eating facilities, bathrooms and offices. In many museums, students will be able to have a special tour to see first-hand how exhibits are created.



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Dear Families,

Our current theme in the Afterschool Math Plus program is the Built Environment.

In this theme students will become architects as they look at their communities, participate in a design session called a charette, create maps of an ideal community, and build a model that will demonstrate their new knowledge and skills. This model will be displayed where families can see the work and celebrate their child's achievement. As soon as we have a date for this event, we will get the word out!

There are many ways you can help your child make the most of this theme:

- Take a walk around the neighborhood to look at the buildings and other structures.
- Make a list of the different kinds of buildings, noting shape and structure.
- Visit a local science museum and discuss the math in the exhibits with your child.
- Go on the internet: there are lots of sites that have information about architecture, careers in math, and fascinating role models.

And be sure to ask your child about the Built Environment math activities! They are exciting and full of interesting information that your child can share with you.

All the Afterschool Math Plus hands-on, minds-on activities will help your child develop essential math skills while having fun. The activities all meet the national mathematics standards, are engaging, and have an added "Plus." The Plus is a series of career and role model activities to broaden your child's ideas about who does math, how math is a part of everyday life, and possible math careers.

Please feel free to ask a staff person about Afterschool Math Plus and your child's participation.

Sincerely,

Queridas Familias,

Nuestro tema actual en el programa Matemáticas Despues-de-Escuela y MÁS es el Built Environment (Ambiente Construido).

En este tema los estudiantes serán arquitectos. Según miran los componentes del ambiente construido, crean mapas de una comunidad ideal, y construyen un modelo que demuestra su nuevo conocimiento y habilidades. El modelo será exhibido donde las familias pueden ver el trabajo y celebrar el logro de su niño/niña. ¡Tan pronto como tengamos una fecha para este evento le dejaremos saber!

Hay muchas maneras en las que usted puede ayudar a su niño/niña a aprovechar este tema:

- Tome una caminata alrededor de la vecindad para ver los edificios y otras estructuras.
- Haga una lista de las diversas clases de edificios que observan, note la forma y estructura.
- Visite un museo de la ciencia local y hable sobre las matemáticas en las exhibiciones con su niño/niña.
- Vayan al internet: hay muchos sitios que tienen la información sobre arquitectura, carreras en matemáticas, y modelos positivos y fascinantes.

Asegúrese de preguntarle a su niño/niña acerca de las actividades del ambiente construido y las matemáticas. Es excitante y contienen mucha información interesante que su niño/ niña puede compartir con usted.

En todas las actividades del programa Matemáticas Despues-de-Escuela y MÁS se usan las manos y la mente y ayudan a su niño/niña a desarrollar habilidades esenciales de las matemáticas mientras se divierten.

Todas las actividades aplican los estándares nacionales de las matemáticas, son interesantes, y tienen algo MÁS. El MÁS es una serie de actividades con modelos positivos que ayudan a ensanchar las ideas de su niño/niña sobre quién participa en las matemáticas, cómo la matemática es parte de la vida diaria, y posibles carreras dentro del campo de las matemáticas.

Por favor, siéntase cómodo/a de preguntarle a nuestros empleados sobre el programa Matemáticas Despues-de-Escuela y MÁS y de la participación de su niña/niño.

Sinceramente,

Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

Who Uses Math? Equity Activity +2:
The Power of a Paragraph

ASM+
EQUITY
ACTIVITY

Who Uses Math?

The Power of a Paragraph

This activity is based on research by Geoffrey L. Cohen, Julio Garcia, Nancy Apfel, and Allison Master. Their study demonstrated that after a brief "self-affirming" writing assignment African American students reduced the racial academic achievement gap by 40%.[1] That's why the activity is called "The Power of a Paragraph!"

1. Collect some short biographies of people who use math and are also from underrepresented groups in math and science. These bios can be found on the Internet, in the library, or in the Afterschool Math Plus themes.
2. Ask students to take turns reading the biographies out loud to the group.
3. Ask students to think about their own math biography and to write a paragraph about themselves and how they use math.
4. Together, create a list of the many ways students use math.
5. Students can volunteer to read their math bios to the group and add to their paragraphs if the list generates some new ideas for them.

Photo by https://unsplash.com/@santivedri on Unsplash

1 Reducing the Racial Achievement Gap: A Social-Psychological Intervention, *Science*, Vol 313, 1 September 2006.

Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

+1 Understanding Scale

+1 Understanding Scale

ASM+ INSTRUCTOR GUIDE

Question
What is scale and why is it important?

Objectives
Students will:

- Use scale to create an accurate picture of themselves.
- Use scale and ratio to explore proportion.
- Create and use a metric measuring tape.

Where's the Math?
When students draw a picture of themselves, they will understand that scale is an important concept in creating an accurate representation. Scale is the ratio between the actual size of an object and the representation of it; for example, a map of the United States is kind of a scale model of the country. Also, by making their own metric measuring tapes, students will better understand the metric system.

Math Skills Developed
- Understanding scale
- Methods for using scale
- Understanding ratio and proportion
- Measurement

Materials
- ☐ Drawing paper
- ☐ Centimeter graph paper
- ☐ Markers, crayons
- ☐ Light cardboard (optional)
- ☐ Clear tape (optional)
- ☐ Colored masking tape
- ☐ Data sheet (paper divided, with one column for the student's actual measurements and one column for scaled measurements)

Handout
- ☐ Metric Measuring Tape Template

Getting Ready
Make a copy of the metric measuring tape handout for each student.

NCTM Math Standards

Content Standards

- **Numbers and Operations:** Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- **Measurement:** Understand attributes such as length and width and select appropriate unit of measurement; understand need for standard units; use customary and metric units; solve problems using scale; and explore ratio.

Process Standards

- **Problem Solving:** Build new mathematical knowledge.
- **Communication:** Organize and consolidate mathematical thinking through communication; communicate mathematical thinking coherently and clearly to peers, teachers, and others; and use the language of mathematics to express mathematical ideas.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.
- **Representation:** Create and use representations to model and interpret physical phenomena.

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Part One: According to Height (15 minutes)


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1. Give students graph paper and (holding the paper in portrait mode) ask them to draw a line four boxes up from the bottom. Then, starting at the line, ask them to draw a picture of themselves from head to feet, with their feet touching the line.
    - For students who are visually impaired, it will be particularly important to be able to locate the borders of the paper as well as the horizontal and vertical midlines to help them orient. To mark the borders and keep the paper steady, place brightly colored masking tape around the sides. Before taping, fold the paper into quarters and then unfold; this will leave creases on the midlines.
2. Collect the pictures and hang them on a wall, arranged by height. Use the line at the bottom of the page to line up the pictures. (It may be helpful to put a piece of masking tape across a wall and ask the students to line up the pictures on the tape.) Be sure to describe the drawings aloud for students with visual and learning disabilities.
3. Ask students to line up according to height. Are they in the same order as the pictures? Discuss why or why not.
    - If a student uses a wheelchair, ask in advance whether they would like to be measured (or lined up) sitting or standing. Determining that a student who uses a wheelchair is the shortest, based solely on the line-up method, can be quite exclusive. Allow the student to guide you toward a positive strategy.
    - Allow students who are blind to use a "head-tapping" strategy; as students line up, let the group know that the student will gently tap their heads to figure out the student's place in line. This allows for student autonomy.
4. Ask students for ideas about how they could draw pictures of themselves that would accurately represent order by height. Explain to students that this is drawing to scale. Where else is scale used? Have they ever used a map? Have they ever made a scale model (car, boat, plane, dinosaur)? What purpose does scale serve on a map? On a model?

Part Two: Literacy and Math Identity Activity on Visualizing Scale (15 minutes)

Tell students you're going to show them *Powers of Ten*, a short documentary film made all the way back in 1968 that will help them understand how scale works.

1. Show the Powers of Ten video by designers Charles and Ray Eames.
2. Give students a few minutes to think about what they've seen, then ask how watching the video made them feel.
3. Then ask why they think the video is set up how it is with each 10 seconds showing a box that is 10 times bigger or smaller than the last one? How does that help them understand scale better than saying 1, 10, 100, 1,000, etc.?

4. Ask them why it might be important that they kept the time at 10 seconds and the scale at 10 times the size? What would probably happen if they made one section longer than 10 seconds or more than 10 times as large?

Part Three: Making Metric Measuring Tapes (15 minutes)

Tell students that now that they've had a chance to visualize what scale means, they can practice it. Give each student a copy of the metric measuring tape template (students can work in small groups or individually). The template provides 10 strips, each 11 centimeters long. Have students cut the strips and tape them together, overlapping where indicated at the end. (Students can color in the 10-centimeter lengths, which will make it easier to see the overlap. This is particularly helpful for students with low vision or learning disabilities.

To make the metric measuring tapes sturdier, students can cover them with clear tape, or they can paste the template onto light cardboard or heavy paper. For visually impaired students, the centimeter and meter marks can be identified by placing glue dots on the various points and allowing them to dry.

Part Four: Drawing to Scale (15 minutes)

1. Hand out centimeter graph paper and explain that students will be drawing another picture of themselves, this time to scale. Ask students to decide on the scale everyone will use, e.g., 1 centimeter on paper = 10 centimeters measured.

2. Have students work in pairs to measure each other, using their metric measuring tapes. They should measure head to feet, shoulder to shoulder, shoulder to fingertips. Students should record their measurements on the data sheet and then use those measurements to create a scale drawing of themselves.

3. Ask students to use these measurements to draw a picture of themselves to scale on the graph paper. Visually impaired students can use their tactile metric tapes to create their scaled drawings.

4. Ask students to collect the pictures, arrange them by height, and line themselves up accordingly. Are the pictures now lined up in order of students' actual height? Ask students to discuss what purpose using scale served in their drawings.

Note to Group Leaders:
Remember to save the metric measuring tapes for the next activity!

Canva

Additional Literacy and Math Identity Activities

If one of these activities better suits your students or the goals of your center or program, replace Part Two with it. Or add these activities to further expand your students' math identities.

- Read one or more of the books that translate the powers of 10 to the page with students and discuss how the book helps them understand the concept of scale:
  - » *Magnitude: The Scale of the Universe*, by Kimberly Arcand and Megan Watzke, both of NASA, uses drawings and infographics to explore and visualize the concept of scale in the universe. (New York: Black Dog & Leventhal, 2017).
  - » *Powers of Ten: A Flipbook*, by Charles and Ray Eames, presents photos of each level along with what you are looking at, one picture per page. (New York: W.H. Freeman Company, 1998).
  - » *Powers of Ten: About the Relative Size of Things in the Universe*, by Philip and Phyllis Morrison. Each page moves inward, reduced at one-tenth the scale of the previous page. (New York: Scientific American Library, distributed by W.H. Freeman, 1984). Available free through the Internet Archive.

- Take students on a virtual tour of the Scales of the Universe exhibit at the American Museum of Natural History in New York. This exhibit makes use of the Hayden Planetarium dome's size as the basis for their visualization of the powers of 10 and the scale of the universe as well as the microscopic. This video is long (31 minutes), so consider showing excerpts or breaking it up over several sessions. After students have watched the video, have them reflect and discuss how the visualization helped them to understand the concept of scale.

Equity

Some students may be sensitive about being the shortest or tallest person in the room. So be sure to arrange from short to tall and tall to short when ordering by height. It is important to address any teasing that might occur about a student's height. Unfortunately, teasing and bullying are facts of life for some students. Any perceived difference can set off teasing and bullying behavior, including about height, weight, gender, race, ethnicity, or class. It is important to be proactive if such behavior occurs.

Reflecting on the Activity

- Did students understand the concept of scale? Was I comfortable in talking about scale, ratio, and proportion?
- Did students of all genders participate equally?
- Did I make sure that the activity was inclusive of students with disabilities?
- Do students need more help understanding the metric system?

Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

+2 Taking Inventory

ASM+
INSTRUCTOR
GUIDE

+2 Taking Inventory

Question
What is the built environment?

Objectives
Students will:

- Work in small groups to list the different features of the built environment.
- Take a walk to observe the elements of the built environment.
- Share ideas and discuss the built environment in a larger group.

Where's the Math?
Students will observe the architectural features and identify patterns, geometric shapes and angles in the built environment and discuss the connection between the form and function of various structures.

Math Skills Developed
- Identifying patterns
- Identifying geometric shapes and angles

Materials
- ☐ Paper
- ☐ Pencils
- ☐ Chart paper
- ☐ Pieces of cardboard and clips to make clipboards
- ☐ Markers

NCTM Math Standards

Content Standards

- **Algebra:** Understand patterns, relations, and functions.
- **Geometry:** Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Process Standards

- **Problem Solving:** Build new mathematical knowledge.
- **Communication:** Organize and consolidate mathematical thinking through communication; communicate mathematical thinking coherently and clearly to peers, teachers, and others; and use the language of mathematics to express mathematical ideas.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

Handouts
- ☐ Geometric Shapes
- ☐ Role Model Biographies

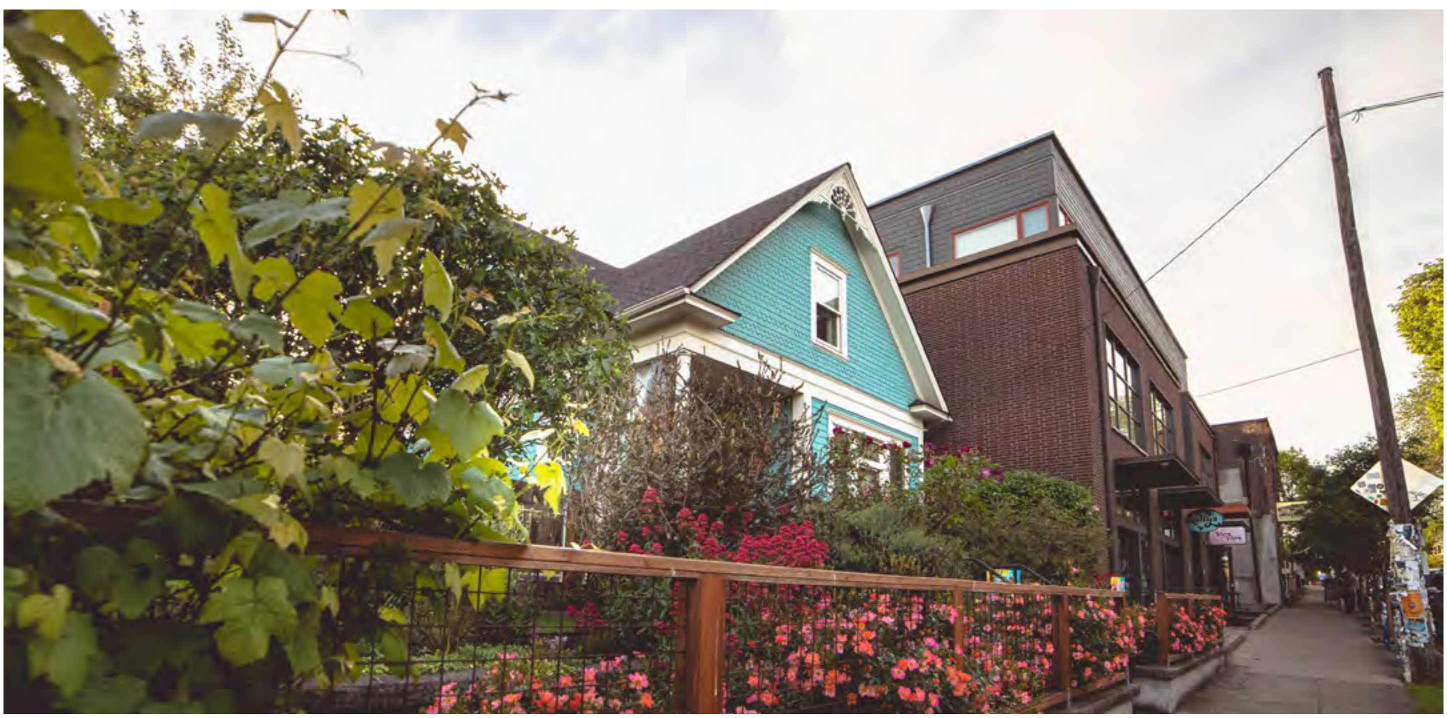

Photo by https://unsplash.com/@cudithemillennial on Unsplash

Getting Ready

Before doing this activity with students, take a walk around the neighborhood to decide where to take students to see the built environment. Make note of any architectural features and the terrain that you would like the students to notice. Be aware of obstacles for students with physical disabilities or who are low-vision/blind. Ask parents/volunteers to be on hand for the walk. A few days before the walk, gather pieces of cardboard and clips so that students can make clipboards to hold the handout and note paper during the walk.

Make copies of the Geometric Shapes handout for each student.

If there are students with visual impairments in your group, you can easily make the Geometric Shapes handout tactile using wax-covered yarn. You can also create a "shape ring" for blind students to carry. The ring would consist of different shapes made of pipe cleaners or wire and labeled. Labels can be created by a Braille labeler or by writing raised glue letters on paper (e.g., "H" for hexagon, "P" for pentagon, etc.).

Part One: Small-Group Investigations (15 minutes)

1. Divide students into groups of four or five. Ask two students from each group to take turns as the recorders and write down ideas and comments during the discussion. It is important to have a good record for the whole-group discussion that will follow.

2. Ask students to discuss these questions:

    - What do you think is meant by the built environment?
    - What types of buildings or structures make up the built environment?
    - Why might different geographic locations have different built environments?
    - Are there components to built environments that are the same everywhere?
    - What do you think is the most important part of the built environment? Why?
    - Think about the shapes in the built environment. Are the form and function of buildings related? How?

3. Remind each group to record their answers and ideas as they discuss the questions.

Part Two: Sharing Findings (20 minutes)

Bring the students together in a large group. Ask each group to report on what they discussed. During the group discussion, ask students to use the "Pass the Microphone" method of taking turns; that is, only the student with the "microphone" (a pencil, a cone of paper, etc.) can speak. This ensures that each person's comment will be listened to and allows students with hearing impairments to easily see who is speaking. As students report, create a list on chart paper of all the elements that make up the built environment. Remember to say the words as you write them to give both auditory and visual cues. Save the list for later reference.

Part Three: Literacy and Math Identity Activity (20 minutes)

1. Ask students how much of what they shared in their discussion was based on where they live or where they come from. Were there ideas that their partners said that sounded like what they've seen in their neighborhoods? Different from their neighborhoods? How many of them have lived more than one place? Are the parts of the built environment where they live now different from places they've lived before? When they see a building or a place from their past—like maybe their elementary school if they're in middle school now or the park they went to when they were really little—how does that make them feel?

2. Distribute the Role Model Biographies handout. Have volunteers read Daniel Glenn's biography aloud.

3. Ask students to write a paragraph about themselves that mirrors Daniel Glenn's biography. Ask them to include where they grew up, their heritage, what they think is important to them, and how it will affect their careers.

4. Daniel Glenn said, "I've discovered that if you want to have a significant impact, it is very difficult to do it any place other than in your own home. If you are rooted in a place, you have a right to make it a better place." Help students reflect on their list of the elements that make up the built environment and which of these elements would make a community a better place. Be sure to include the need to make all public spaces accessible.

Part Four: Introducing the Walk (5 minutes)

Bring students together in a large group and explain they will be going for a neighborhood walk to look at the built environment. Review the list they made. Explain that they also will be looking for shapes in the built environment. Give each student a copy of the Geometric Figures Handout and briefly review the shapes. (Two-dimensional shapes: square, rectangle, triangle, parallelogram, rhombus, trapezoid, pentagon, hexagon, ellipse, and circle. Three- dimensional shapes: cube, sphere, column, cone, tetrahedron). Also give students the cardboard and clips so that they can make a clipboard for note-taking during the walk. Remind everyone to bring a pencil.

Note to Group Leaders:
If it is not possible to take a walk outside, have students go on a virtual walk of an architectural website like ThoughtCo.'s Great Buildings and Structures in Architecture or one of the tours on Google's Arts & Culture Places page.

Part Five: Taking Inventory (20 minutes)

As students take the walk, ask them to take an inventory, noting on their clipboards the buildings and structures. Ask students to think about the details of the built environment. Are the structures unique? Repeated elsewhere on the walk? Tall? Wide? Close together or far apart? What else do they notice? What shapes are the buildings? (Students can use the handout as reference.) Similar to buildings in their neighborhoods or that they go to often or not? Is the environment accessible for people with disabilities? Do buildings have ramps as well as stairs? Are there corner cuts on the streets?

Another way to take inventory is to have students photograph buildings and other interesting aspects of their built environment, e.g., fences, sidewalk patterns, parks. If you have one or more cameras available, students can take turns photographing something in the built environment that is of special interest to them. Use the photographs along with notes, drawings or notations on the handout that students have made to document their observations of the built environment. Pair blind students with sighted students for this visual observation activity. Visually impaired students should have the opportunity to record the observations using any method that is helpful to them, e.g., marking tallies on a tactile shape sheet, bringing a recording device, using a voice-to-text or AI surroundings description app on a smartphone or tablet, or asking a partner to assist.


Photo by https://unsplash.com/@jair0g0nza on Unsplash

Part Six: What Did We Observe? (15 minutes)

1. Bring the group together to discuss what they saw on the walk. Refer to the list they made at the beginning of the activity. Would students like to add anything to the list? Ask students if they have any unanswered questions. If so, ask the group to brainstorm some answers.

2. Make a list of shapes and structures that students observed. Ask students whether some shapes were observed more frequently. Why do they think this is so? How are the shapes used? For support? To enclose an area? As a foundation? Does the shape of the building matter? Compare structures to each other for size, shape and function.

3. Ask students what was the favorite part of their built environment? What would they change? Why? Ask if there were any particular challenges in the environment for students with disabilities? What were they and how could they be improved? Did anyone notice any special features designed to assist persons with disabilities such as ramps or curb cuts?

4. Ask students if there are any famous structures that use the shapes they saw on their walk. For example, the pyramids in Egypt (polyhedron), the Gateway Arch in St. Louis (parabola), or the World's Fair Unisphere (sphere) in Flushing Meadow Park.

Additional Literacy and Math Identity Activities

If one of these activities better suits your students or your center or program goals, replace Part Three with it. Or add these activities to help build your students' math identities.

- Show students segments from the Denver Museum of Nature & Science's discussion of the indigenous film "From Earth to Sky," which presents the work of seven Indigenous North American architects. The conversation is moderated by Mervyn Tano, president of the International Institute for Indigenous Resource Management, and features Daniel Glenn and Tammy Eagle Bull, the first Indigenous woman licensed as an architect in North America, discussing their inspiration to become architects, working with Indigenous communities, community participation and service, and some of their architectural projects.
- Take a virtual tour of famous structures. Be sure to look at pictures of the structures built by this theme's role models: Karen Braitmayer, Daniel Glenn, Zaha Hadid, Francis Kéré, and Maya Ying Lin. Make a list of ways that their structures are unique.
- Go to the Origami Resource Center for paper polyhedron patterns that students can use to create 3D geometric folded shapes.
- Have students interview or survey local community members (other students attending programs at the center, their caregivers and families, and/or their teachers, coaches, and other adults in their lives) about what they want to see in their community. Share these findings with the group and add them to the list of elements of the built environment.
- Visit the library to look for books about architecture by David Macaulay. Some of the titles are:
  - *Cathedral: The Story of Its Construction* (Boston: Houghton Mifflin Company, 1974. Revised full color edition 2010).
  - *City: A Story of Roman Planning and Construction* (Boston: Houghton Mifflin Company, 1983).
  - *Pyramid* (Boston: Houghton Mifflin Company, 1977).

Equity

Small group activities encourage cooperative learning experiences. These experiences help all students learn to communicate, respect each other's ideas, and spark each other's creativity.

Reflecting on the Activity

- Did all students contribute ideas in the large discussions?
- Did students enjoy the walk?
- Was there anything I could do to improve their experience?

Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

+3 Measuring Up

+3 Measuring Up

ASM+ INSTRUCTOR GUIDE

Question
How can we measure and map our environment?

Objectives
Students will:

- Design and use a measurement tool.
- Convert measurements using a specific scale.
- Collect and use data to construct a two-dimensional map.

Where's the Math?
Students measure features and collect data from the built environment to create a two- dimensional map. Ratio, proportion, and scale are key elements in creating an accurate representation of the built environment.

Math Skills Developed
- Measurement
- Methods for using scale
- Representing information on a map
- Construction of a two-dimensional map
- Interpretation of a map using a scale, which is described in a key/legend

Materials
- ☐ A selection of maps
  If available, raised relief (3D) maps are wonderful in that they provide both visual and tactile cues for all students and especially for those with different learning modes or sensory disabilities. Also, inexpensive Braille maps are available at www.independentliving.com.
- ☐ Metric measuring tapes—one for each group
- ☐ Clothesline rope (approximately 8 meters or 25 feet—one for each group)
- ☐ Masking tape in two colors
- ☐ Blank paper
- ☐ Chart-size centimeter graph paper
- ☐ Compass
- ☐ Protractor
- ☐ Pencils
- ☐ Magnifying glasses for students with low vision

Getting Ready
A few days before this activity, take a walk through the afterschool center to select a space for students to measure. A gymnasium, lunchroom, or other open area is a good choice.

NCTM Math Standards

Content Standards

- **Numbers and Operations:** Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- **Measurement:** Understand attributes such as length and width and select appropriate unit of measurement; understand need for standard units; use customary and metric units; solve problems using scale; and explore ratio.
- **Data Analysis:** Collect data using observations and represent data accurately.
- **Geometry:** Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Process Standards

- **Problem Solving:** Build new mathematical knowledge.
- **Communication:** Organize and consolidate mathematical thinking through communication; communicate mathematical thinking coherently and clearly to peers, teachers, and others; and use the language of mathematics to express mathematical ideas.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.
- **Representation**: Create and use representations to model and interpret physical phenomena.

Photo by https://unsplash.com/@rrruthie on Unsplash

Part One: Literacy and Math Identity Activity (15 minutes)

Explain to students that you'll be taking a virtual tour of the buildings and architectural design work of Zaha Hadid, an Iraqi-British architect, artist and designer, who was the first female architect to receive the Pritzker Architecture Prize and the Royal Institute of British Architects (RIBA) Royal Gold Medal.

1. Bring up Google Arts & culture's Zaha Hadid exhibit and walk students through the photos. Have a volunteer read the captions.
2. Bring up the Google Arts & Culture editorial, A Virtual Tour Of Zaha Hadid's Most Iconic Buildings. Also have a separate tab open to Google Maps to pull up these buildings on the map as you walk through the photos.
    - Have volunteers read the photo captions out to the group
    - Ask what students think about how the building interacts with the environment around it.
    - Pull up the building on Google Maps and show the students the zoom to the building from a high level (as a reminder of the Powers of Ten activity). Ask students to describe the scale of the building in relation to those around it. Is the building larger and dominating the area? Does it fit in? Does it look like the buildings around it? Ask students how seeing the scale of the building in context of its surroundings changes their thoughts.
3. Keep one of these maps open to serve as an example in the next part of this activity.

Part Two: Thinking about Maps (15 minutes)

Explain to students that they will measure a room, record the information, and use what they learned about scale to make a two-dimensional map. Share some examples of maps with students. If a raised relief map is not available for a visually impaired student, two-dimensional maps can be made tactile using sand, glue, or wax-covered yarn.

Ask students to think about the different parts of a map. Some questions to think about include:
- What are the components (different parts) of any map?
- What components must always be included?
- What purpose does the map serve?
- How can distance be measured on the map?

Find the key/legend on the map. Ask students what they think it represents. How does the key make the map more useful and easier to interpret? Are there other features on maps that help us read them more completely? Discuss the ways maps are used. Even if the map has not been made tactile for visually impaired students, be sure that the map key is, so that the students have an idea of the scale being used. Remind students about their drawings of themselves to scale. Tell students that as part of this activity they will create a map to scale.

Part Three: Making a Tool (15 minutes)

1. Divide students into groups of three to four and show them the room they will be measuring.
2. Ask for volunteers to be measurers and others to record the measurements. Explain that everyone will be switching jobs to experience both roles.
3. Give students the metric measuring tapes they made in Activity 1 and show them the rope and masking tape. Ask if they have any ideas about how to create a tool to measure distance. If none of the students suggests it, bring up the idea of turning the rope into a measuring tool.
4. Using a metric measuring tape, ask students to use one-color masking tape to mark off one-meter lengths along the rope. Then, have students measure and mark off 10-centimeter lengths using the other color tape. For students who are visually impaired, knots can be made in the rope at meter marks, with tape at the half-meter marks. These will present as two distinct textures.

Part Four: Small-Group Investigations (20 minutes)

Have each group decide on an area of the room to measure. Then using their new tool, have students measure the dimensions of their area and record them on a sheet of paper. They may choose to make a sketch of the room and label the sketch, or they may make a list of the walls, windows, and other features and chart the information. Remind students to include as many of the architectural features of the room as they can identify. (Windows, doors, pillars or columns, support beams, etc.) Encourage students to measure width, height, and depth of each feature. Be sure that students who utilize wheelchairs can work with features at their level. Students can also use the chair's wheels as measuring tools. If the student places a piece of colored tape at the top of the wheel and then begins to travel along the metric measuring tapes (or rope), they can see what distance is traveled during one rotation of the wheel. Measuring longer distances is simply a matter of counting the rotations and multiplying. This may make measuring easier than with a tape or rope.

Part Five: Whole-Group Investigation (20 minutes)

1. Explain to students that they will be using a large sheet of centimeter graph paper to create a map of the room they measured. To do this they will need to create a scale that will allow them to fit the map on the centimeter paper.

2. Have students decide on a scale for the map. Suggest looking at the longest measurement to see if using a scale of one centimeter on the map to one meter in the room gives an accurate representation. Will it fit on the graph paper? Help students see if it will or won't work. If that scale doesn't work, try another scale, e.g., one centimeter equal to two meters or one centimeter equal to half a meter.

Note to Group Leaders:
Try out some scales beforehand so you can guide students to a scale that works.

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3. Remind students about the map key/legend they saw on the sample maps and have them create one for their map. Together, decide on symbols for doors, windows, and other architectural features and include them in the legend. For example, a dotted line could represent a window and a rectangle with a small circle for a doorknob could represent a door.

4. Ask students to choose a starting point on the grid paper. Draw lines for the walls from that point. Measure along the lines and insert the symbols for other features. These symbols should also be drawn to scale.

5. Show students how to use a compass or protractor to make a curved line or circle if there are curved or round features to the room.

6. A scale drawing shows how long each of the walls is and the location of the different features. Ask students if there is anything else that they might want to include in the map.

7. Congratulate students for having successfully created their own two-dimensional maps.

Additional Literacy and Math Identity Activities

If one of these activities better suits your students or your center or program goals, replace Part One with it. Or add these activities to help build your students' math identities.

- Have students explore the world of maps, either online or at the library.
- Read *The World Is Not a Rectangle: A Portrait of Architect Zaha Hadid* by Jeanette Winter with students. It tells the story of Hadid growing up in Baghdad, Iraq, dreaming of building cities, and succeeding in her architecture career despite obstacles. (San Diego: Beach Lane Books, 2017).
- Have students read *Zaha Hadid (Volume 31 of Little People, BIG DREAMS)* by Maria Isabel Sanchez Vegara and Asun Amar. Part of a biography series for kids, this book tells Hadid's story from her childhood in Iraq through illustrations and historical photos. (London: Frances Lincoln Children's Books, 2019).
- Share a copy of *Measuring Penny* by Loreen Leedy with the group. In this story, Lisa measures her dog Penny using all kinds of found objects. Make a list of things that can be used as measuring tools. (New York: Henry Holt and Co., 2000).

Equity

Every student can take an active role in this activity. Students should be encouraged to listen respectfully to each other's ideas. Students who identify or present as girls traditionally have fewer experiences that increase their spatial relations skills, which are needed to support math and science skills. This measuring activity helps all students gain these skills.

During the measuring activities, make sure that students of all genders take turns, equally doing the hands-on measuring and recording. Often, if left to assign tasks on their own, students fall into stereotyped patterns.

Reflecting on the Activity

- Did all students participate in the activity? Did students of all genders share the roles of measuring and recording equally?
- Did students have enough space to measure?
- Were students able to draw a map to scale?
- Do I need to review Activity One: Understanding Scale?

Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

+4 Planning the Ideal Community

ASM+
INSTRUCTOR GUIDE

+4 Planning the Ideal Community

Question
What are the elements of an ideal community?

Objectives
Students will:

- Decide on components of an ideal community.
- Consider the relationship between form and function for structures.
- Share their plans and discuss the logic behind their design.

Where's the Math?
Planning is a very mathematical activity! Students will use the skills they learned about scale and measurement, as well as information they gathered in their neighborhood walk, to plan an ideal community. Students will estimate the relative size of buildings and will engage in problem-solving and communicating their ideas to each other.

Math Skills Developed
- Problem-solving
- Representing mathematical data
- Interpreting data

Materials
- ☐ Pencils
- ☐ Paper
- ☐ Tape
- ☐ Drawings and maps from previous activities

Handout
- ☐ Design Questions
- ☐ Role Model Biographies handout

Getting Ready
Make copies of the Design Question handout for each student.

NCTM Math Standards

Content Standards

- **Algebra:** Describe, extend, and make generalizations about geometric and numeric patterns; and model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.
- **Data Analysis & Probability:** Propose and justify conclusions and predictions that are based on data.
- **Geometry:** Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Process Standards

- **Problem Solving:** Apply and adapt a variety of appropriate strategies to solve problems.
- **Reasoning and Proof:** Select and use various types of reasoning and methods of proof.
- **Communication:** Communicate mathematical thinking coherently and clearly to peers, teachers, and others.
- **Representation**: Create and use representations to organize, record, and communicate mathematical ideas; and use representations to model and interpret physical, social, and mathematical phenomena.

Photo by https://unsplash.com/@mirandanef on Unsplash

Part One: Literacy and Math Identity Activity (15 minutes)

Explain to students that you're starting this activity by looking at the work of another architect, educator and social activist, Francis Kéré, who was born in Burkina Faso in 1965, and is the first black and first African architect to win the Pritzker Prize.

1. Show students the PBS News Hour clip, "First Black Winner of Pritzker Architecture Prize discusses his community-focused designs." Ask students to listen to Kéré as he speaks about his work and note what seems to be most important to him in creating his architectural designs, and what about his process is different.

Note to Group Leaders:
The site includes a full-text transcript, which you can provide to students who are deaf or hard of hearing.

2. Ask students to share what they noted as they watched the video.

3. Bring up either the Arch Daily feature, "Francis Kéré Receives the 2022 Pritzker Architecture Prize," or the BBC News interview, "Diébédo Francis Kéré: The first African to win architecture's top award" and have volunteers read the text to the group while looking at the photos of Kéré's work. Ask students to think about what ideas of his they might want to include in their list of elements of a built environment.

4. Have students share their responses to Kéré's designs and ideas. Add elements to the list of a built environment that they created in the earlier activity.

Thinking about the Ideal Built Environment (20 minutes)

1. With students, review the list of elements of a built environment. Remind them that they have had experience with observation measurement, scale, data analysis, and mapping.

2. Have students take out their Role Model Biographies handouts (or redistribute them). Have a volunteer read Karen Braitmayer's biography aloud.

3. Share photos and excerpts from the 2021 Architectural Digest profile of Braitmayer and from the website for her design firm, studio pacifica.

4. Ask students which of the elements they listed before would be helpful for someone who uses a wheelchair, someone who is pregnant or pushing a stroller, or for very young children or older adults. Use sticky notes, markers of different colors, plus signs, or shapes to mark the elements of the built environment that make it more accessible. Tell students that these are some of the considerations they'll need to take into account while they create their ideal built environment.

Photo by https://unsplash.com/@mannnapat on Unsplash

5. Give students the Design Questions handout and ask them to think about their ideal built environment. What would it be like? Would it be urban or suburban? Would it have schools? Would there be streets? Parks? What would the houses look like? Would there be trees and other greenery? Chart their responses.

    - During the discussion, ask students to be sure that only one student speaks at a time and is identified by raising a hand. This process is beneficial to all students and can help a student who is deaf or hard of hearing focus on the sound and/or read lips.

6. Ask students to talk about the various features of the built environment that are mentioned. Why are they important? What should they look like? Do the forms of the structures matter? Where should they be in relation to each other?

7. Ask students to organize the list of structures and architectural features into clusters that make sense to them. For example, they may decide to cluster a school, park, and library together. Or, they may cluster residential buildings, parks, supermarkets, and a hospital. Chart a list of the structures in each cluster. Use a separate piece of chart paper for each cluster and write the structures/features in large lettering. This will be helpful for all students and especially for students with low vision and learning disabilities.

8. Ask students to think about the arrangement of the clusters in the ideal community. Should the school be located next to the department of sanitation? Should the hospital be located next to an outdoor theatre where loud music is played? Ask students to organize the location of these clusters in a way that makes sense to them and explain their reasoning.

Part Three: Drawing a Rough Sketch (20 minutes)

1. Ask students to work in groups of four. Explain that each group will be a design team responsible for creating a rough sketch or map of one of the clusters that they described in the group discussion. Explain that when their sketches are connected, an ideal environment will be represented. (See chart below.) Ask students to keep in mind which structures will ultimately be adjacent to their cluster.

! *Note to Group Leaders:*
Be sure to have the list of structures available for each group to use in their planning.

A Cluster Sketch	B Cluster Sketch	C Cluster Sketch
D Cluster Sketch	E Cluster Sketch	F Cluster Sketch

2. Remind students that although they are making a rough sketch or map that they should keep scale in mind and that structures should be drawn in proportion to each other.

    - To ensure that all students contribute to the small-group discussion, suggest that the design teams use a round robin technique to make recommendations about the locations of the structures.
    - If it appears that one student is dominating the sketching, suggest that each student be the principal architect for one structure, thereby ensuring opportunities for each person.
    - If students who are visually impaired are present, encourage their teammates to guide the student to the appropriate spot on the drawing, but allow the student to do the actual sketch if they choose. Wax-covered yarn also can be used so that visually impaired students can detect the boundaries between structures. Or have students use a voice-to-text application with an AI art generator in a digital illustration tool to contribute their building sketch.

Part Four: Sharing Findings: (10 minutes)

Have students gather into a large group and have each design team present their sketches. As they present, they should describe their process and rationale for the location of the different structures. Organize the sketches according to the cluster chart and hang the sketches on the wall.

The following questions may be helpful in the discussion.

- What kinds of buildings are grouped together?
- Did you move structures during the process? Why?
- Were parks and other green places included? Why are these important?
- How were form and function used in the plan?
- What was most challenging? What would you do differently next time?
- What elements, if any, did you include that were inspired by Kéré's work or Braitmayer's work?

Additional Literacy and Math Identity Activities

If one of these activities better suits your students or your center or program goals, replace Part One with it. Or add these activities to help build your students' math identities.

- Search online for an "ideal community." List the components architects and civil engineers consider essential. Compare with the list made by the students. Many cities are working hard to design communities that are accessible to persons with disabilities.
- Share The Guardian's "What would a truly disabled-accessible city look like?" (or a more recent article or an article about a community more local to you) and have students discuss what elements from the cities described sound like they would benefit an ideal community.
- Read *Building Liberty: A Statue is Born* by Serge Hochain to follow the creation of the Statue of Liberty from design to installation. Compare this story to the Vietnam War Veteran's Memorial history. (See Career and Role Models Connections section.) How are the stories the same? How are they different? (Washington, DC: National Geographic Society, 2004).

Equity

The concept of an ideal community will be different for each student. They all bring their experiences to the project. Encourage students to respect and validate each other's contributions to the ideal community.

Be aware that students might express their anxieties in this activity, e.g., in structurally excluded and disinvested communities, students might be concerned with creating a community organizing space, a food pantry, or a community garden. Or if a recent police violence or active shooter incident has occurred locally or been prominently discussed online or in the news, students might express anxiety about the safety of school buildings or public spaces.

Reflecting on the Activity

- Did all students participate in planning?
- Were students able to sketch a map?

Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

+5 Mapping the Ideal Community

+5 Mapping the Ideal Community

ASM+ INSTRUCTOR GUIDE

Question
How can we create a map of our ideal community?

Objectives
Students will:

- Work in groups to communicate ideas on their ideal community.
- Connect previous learning to create a blueprint of the ideal community that will be used to create a model built to scale.
- Use standard metric units to create the map.
- Share their ideas with the larger group and receive feedback.

Where's the Math?
Students use measurement, scale and proportion, geometry, and algebra as they create a map of their ideal environment. They also communicate and problem-solve as they create their final design, in a work session called a charrette, which will lead to their making a scale model at the science museum.

Math Skills Developed
- Measurement
- Ratio, scale, and proportion
- Representing data using tables and charts

Materials
- ☐ Pencils
- ☐ Paper
- ☐ Chart-size centimeter graph paper
- ☐ Metric measuring tapes
- ☐ Markers
- ☐ Compass
- ☐ Protractor
- ☐ Rough sketch of the ideal community (from Activity 4)
- ☐ Wax-covered yarn
- ☐ Pieces of hook-and-loop tape

Handout
- ☐ Role Model Biographies handout

Getting Ready
Mount the student sketches of the ideal community in the room where the students will work.

NCTM Math Standards

Content Standards

- **Data Analysis & Probability:** Collect data using observations, surveys, and experiments; represent data using concrete objects, pictures, and graphs.
- **Measurement:** Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute; understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems; solve problems involving scale factors, using ratio and proportion;
- **Algebra:** Understand patterns, relations, and functions.
- **Geometry:** Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Process Standards

- **Problem Solving:** Solve problems that arise in mathematics and in other contexts; and apply and adapt a variety of appropriate strategies to solve problems.
- **Communication:** Organize and consolidate mathematical thinking through communication.
- **Representation**: Create and use representations to organize, record, and communicate mathematical ideas.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

Jessica Scranton for FHI 360.

Part One: Introduction (5 minutes)

Bring students together for a discussion. Tell them that they will be creating a blueprint, which is a type of map that architects use to plan a building, in a design session called a charrette. They will use the blueprint as a guide when they build a three-dimensional model at the science museum. Explain that the word "charrette" originally meant an architectural student's final work representing what they had learned but now has come to mean an intense period of design activity in a group to meet a deadline in fields like architecture, landscape architecture, industrial design, interior design, or graphic design.

Part Two: Literacy and Math Identity Activity (20 minutes)

Explain that often a charrette includes the people who live and work in the area being designed, so the local neighborhood residents or workers participate in the design process.

Share that they are going to hear once again from a working architect about how they approach this process of incorporating the community, the environment, and the ways that a space will be used as they create their work.

1. Have students take out their Role Model Biographies handout (or redistribute them). Have a volunteer read Maya Lin's biography aloud.
2. Show students the National Portrait Gallery's video, One Life: Maya Lin, In Her Own Words. Ask students to listen for what Lin thinks about in her process as she's creating art and architectural designs.
3. Have students talk a few minutes about how the video made them feel and what they noticed about how Lin approaches her projects.
4. Share additional images of Lin's work found online:
    - The Academy of Achievement includes several photos of her work in her profile as a winner of the Presidential Medal of Freedom (2016), and the National Portrait Gallery's One Life exhibition, which included the short film above, has several photos of her work as well.
5. Ask students what ideas of Lin's they can incorporate in their ideal communities as they start to finalize their designs. Include any ideas on their list of the elements of a built environment.

Part Three: Reviewing Ideas (15 minutes)

1. Remind students that in this theme, they have had experience with scale, measurements, mapping, observation and data analysis. Explain that they will use all these skills to create a blueprint of the ideal community.
2. Review the sketches of the ideal community (from Activity 4). Ask students if they are satisfied with the general plan. Encourage them to sketch in any changes or adjustments they think will improve the plan. Encourage students to think about all members of a community. Revisit the Design Questions from Activity 4 and ask students to also consider:
    - What type of natural environment is the ideal community located in?
    - What natural resources are part of the environment?
    - Are there services that you would like to include in your ideal community that do not exist in your current community?
    - How do people get around in your community?

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- Where do people live in your community?
- Are there open spaces set aside in your ideal community?
- Are there accessibility needs to be addressed?
- Are there facilities for older citizens?
- Are young children and their needs considered in your ideal community?

3. Remind students that the blueprint must be drawn to scale and that everyone must use the same scale because the drawings will be joined at the end of the activity. Ask students to agree on a scale for the map. (They may have to try out a few before they decide.)
   - Be sure to show students the grid paper so that they have an idea of the scale to which they are drawing. Describe the grid paper aloud (e.g., "The paper is __ cm. long by ___ cm. wide and the boxes are ___ cm. on each side").

Part Four: Creating the Blueprint (25 minutes)

1. Have students form into their Design Teams. Distribute large sheets of grid paper and the rough sketches students created. Ask each design team to mark the grid paper with the corresponding letter from their cluster in the top left corner of the paper (see Activity 4).
2. Ask students to leave a border two-grid-boxes wide around the edge of the paper. When the maps are joined at the end of the activity, these borders will be streets between the areas. Temporarily marking the boundaries of the workspace with wax-covered yarn will be very helpful for students with spatial or visual challenges.
3. Ask each design team to transfer their rough sketches onto the grid paper. Remind them to:
   - Use words and symbols to label all architectural features (for example, buildings, sidewalks, windows, doors, fences and gates, etc.)
   - Indicate symbols for features in the map key/legend.
   - Indicate scale.
   - A handheld Braille labeler, which embosses Braille letters onto vinyl labeling tape, can be used to label the various features. Check out www.braillebookstore.com and other online retailers for inexpensive options. Students can also cut pieces of hook-and-loop fastener tape into different shapes on their maps to create tactile markers.
4. Encourage students to use the compass and protractor to create angles within structures and curved features.
5. When design teams have finished, join the blueprints. Remember to join in the established order.

A Cluster Sketch	B Cluster Sketch	C Cluster Sketch
D Cluster Sketch	E Cluster Sketch	F Cluster Sketch

Part Five: Sharing Findings: (10 minutes)

Ask students for feedback on their blueprint. Are they satisfied with the plan? Are there additions or changes they would like to make? Save the blueprint to use at the science museum when creating the charrette.

Additional Literacy and Math Identity Activities

If one of these activities better suits your students or your center or program goals, replace Part Two with it. Or add these activities to help build your students' math identities.

- Read *Maya Lin: Artist-Architect of Light and Lines* by Jeanne Walker Harvey and Dow Phumiruk. A beautifully-illustrated book, it tells Lin's story from a child observing the nature around her to designing the Vietnam Memorial and her adult life as an architect and artist. (New York: Henry Holt and Co. Books for Young Readers, 2017.) For an accessible biography for older students or more advanced readers, try *Maya Lin: Thinking with Her Hands* by Susan Goldman Rubin. (San Francisco: Chronicle Books, 2017).
- Share a copy of *The Librarian Who Measured the Earth, Vol. 1* by Karen Lasky. It tells the story of Eratosthenes, a librarian in Ancient Greece who figured out a way to measure the circumference of the earth using mathematics (New York: Little, Brown & Company, 1994).
- Tell students to imagine that they are submitting their plan for the ideal community to the state department of community development. Ask students to write a "proposal" to attach to their blueprint that explains why each component is important and why the community is best served with this arrangement. If students interviewed or surveyed community members as an additional activity during the Taking Inventory section, encourage them to include some of this data in their proposal. If not, students can complete interviews or surveys now to support their proposals.

Equity

The design of the final map takes creativity, thought, and a certain amount of risk-taking. Students need opportunities to take risks in a supportive educational environment, especially girls and other students who traditionally have not been supported to take risks.

Reflecting on the Activity

- Do any students still need help with their mapping skills?
- Are the students satisfied with their design?
- Were everyone's ideas discussed? Respected? Included?

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Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

+6 Career and Role Model Connections

Career and Role Model Connections

The role models in this theme are/were architects who think that buildings and other structures need to be more than functional—they need to be pleasing to see and they need to have social relevance! Have students read their short bios and try some of the suggested activities.

1. **Karen Braitmayer** designs commercial, residential, and institutional buildings, and is a consultant on accessibility to projects across the country. As an architect and wheelchair user, she knows first-hand about the importance of designing buildings that are accessible for everyone.

2. **Daniel Glenn** is an architect whose focus is called "social architecture." Glenn grew up in Montana and is proud of his Crow tribal heritage. He returned to Montana because he felt that the most important place to have an impact is your home locale. He designed the Little Big Horn College campus on the Crow reservation. The project gives the architect, who has designed and researched socially relevant architecture around the globe, an opportunity to have an impact on his community.

3. **Zaha Hadid** was an Iraqi-British architect, artist, and designer who became arguably the most famous woman architect in the world before her untimely death at age 65. Her major works included international sites like the London Aquatics Centre for the 2012 Olympics, the Broad Art Museum, the Guanghzhou Opera House, and the Daxing International Airport. The Guardian called her the "Queen of Curves" based on the sweeping geometrical forms used in several of her designs.

4. **Diébédo Francis Kéré** is an architect whose work focuses on sustainability, local materials, and community collaboration. Born in Burkina Faso and based in Germany, Kéré is the first black and African architect to win the Pritzker Architecture Prize (in 2022). His work includes several schools in his home village of Gando, Burkina Faso, conceived of and built with the help of the community using locally available materials better suited to the environment than the traditional concrete.

5. **Maya Lin** designed the Vietnam Veterans War Memorial in Washington, DC when she was only 21! The Vietnam Veterans Memorial Fund announced in 1980 that it would sponsor a nationwide competition to design a memorial honoring those who had served in the Vietnam War. Nearly 1,500 proposals were submitted, but Maya Lin (a young architectural student at the time) submitted the winning design.

Make copies of the biographical information on Karen Braitmayer, Daniel Glenn, Zaha Hadid, Francis Kéré, and Maya Lin for students. Ask for a volunteer to read the biographies aloud.

Additional Literacy and Math Identity Activities for this theme:

- If students are interested in finding out more about these interesting role models, have them do research online and report back to the group.
- Look up Karen Braitmayer online. Discuss how being a wheelchair user influences her work.
- These role models think that the buildings they design should be attractive and have social relevance. Architecture combines math, science, and art. Make a list with students of careers that combine math with another subject. Students may need help making this list and finding out about the careers. Together think about ways to find out this information (for example, information may be available online or at the local library).
- Share a copy of *Shigeru Ban Builds a Better World* by Isadoro Saturno and Stefano Di Cristofaro with students (Miami: Tra Publishing, 2023). Ask them to discuss how Shigeru Ban's work as an architect is similar to or different from the other architects they've learned about and discussed in this theme.
- Have each student choose one of the biographies in *101 Black Women in Science, Technology, Engineering, and Mathematics* by L.A. Amber to read and present to the rest of the group. (Independently published, 2020).
- Read "Shaping Contemporary Indigenous Design: An Interview with Sam Olbekson" from the Harvard University Graduate School of Design with students. Ask them to compare what Olbekson discusses with what they learned about and heard from Daniel Glenn. How is there approach to work similar or different?
- Share one of these stories of women computers, mathematicians, scientists, engineers, and astronauts based on your students' ages and reading levels:
  - *Classified: The Secret Career of Mary Golda Ross, Cherokee Aerospace Engineer* by Traci Sorell and Natasha Donovan (Minneapolis: Millbrook Press, 2021).

» *A Computer Called Katherine* by Suzanne Slade and Veronica Miller Jamison (New York: Little, Brown Books for Young Readers, 2019).
» *Counting on Katherine: How Katherine Johnson Saved Apollo 13* by Helaine Becker and Dow Phumiruk (New York: Henry Holt and Co. Books for Young Readers, 2018).
» *Galaxy Girls: 50 Amazing Stories of Women in Space* by Libby Jackson (New York: Harper Books, 2018).
» *Hidden Figures: The True Story of Four Black Women and the Space Race* by Margot Lee Shetterly and Laura Freeman (New York: HarperCollins, 2018).
» *Hidden Figures Young Readers' Edition* by Margot Lee Shetterly (New York: HarperCollins, 2016).
» *Katherine Johnson: Ready-to-Read Level 3 (You Should Meet)* by Thea Feldman and Alyssa Petersen (New York: Simon Spotlight, 2017).
» *Mae Jemison: Ready-to-Read Level 3 (You Should Meet)* by Laurie Calkhoven and Monique Dong (New York: Simon Spotlight, 2016).
» *Path to the Stars: My Journey from Girl Scout to Rocket Scientist* by Sylvia Acevedo (New York: Clarion Books, 2020).
» *Reaching for the Moon: The Autobiography of NASA Mathematician Katherine Johnson* by Katherine Johnson (New York: Atheneum Books for Young Readers, 2020).

Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

+7 Handouts

Built Environment Role Model Biographies

ASM+ HANDOUT

Karen Braitmayer
"As an architect, I focus on accessible design"

As an architect and wheelchair user, Karen Braitmayer knows first-hand about the importance of designing buildings that are accessible for everyone. Braitmayer designs commercial, residential, and institutional buildings, and is a consultant on accessibility to projects across the country.

Braitmayer graduated from Rice University with a degree in behavioral science. However, with encouragement from her father to explore other fields, she took some tests that revealed an aptitude for architecture. As soon as she started a Master in Architecture program at the University of Houston in Texas, Braitmayer knew she had made the right career choice. She loved the work from the very first day.

One of her first bosses encouraged Braitmayer to bring her own experiences as a wheelchair user into her design work by creating barrier-free spaces. In 1993, she formed a partnership with a fellow architecture student from Houston. Their company, located in Seattle, Washington, is called Studio Pacifica, and they design audio studios as well as all types of buildings. Studio Pacifica offices are located in a marina in Seattle.

Braitmayer is married to a marine mechanic, and she and her husband share their love of sailing with their daughter. In addition to her work in architecture, Braitmayer advises local review committees on accessibility free of charge. She has helped to make the Seattle Seahawks stadium and the Seattle Civic Center fully accessible.

Read more about Karen Braitmayer in this *Architectural Digest* profile from December 2021.

Daniel Glenn
Bringing Architecture Back Home

While much of architecture deals with structures that please the eye, Daniel Glenn has devoted himself to designing buildings that also satisfy the soul through social architecture.

He grew up in Montana and is proud of his Crow heritage; his parents are enrolled members of the Crow tribe. In Glenn's fifth year of architecture school, he had an internship with a Boston firm that specialized in affordable housing. The work hooked Glenn. His thesis project was a ranch cooperative in Nicaragua using traditional techniques, such as adobe modified to withstand earthquakes.

Glenn earned a master's degree in architecture from MIT specializing in Third World housing, one of just three U.S. citizens in a class of 60. Glenn has continued to design affordable housing for people of many cultures.

After school, Glenn returned to Montana because he felt that the most important place to have an impact is your home locale.

Glenn worked on the internationally recognized design of the Little Big Horn College campus on the Crow reservation, giving him an opportunity to impact his community.

But Glenn said while his heritage is important to him, he is an architect and professor of many dimensions. As he puts it, "I am not solely a Native American architect. I am an architect who enjoys teaching students about the importance and beauty of the social-political and multicultural aspects of architecture projects."

Read more about Daniel Glenn on his design firm's website, 7 Directions Architects / Planners.

Zaha Hadid
"There has been tremendous change over recent years and we will continue this progress."

Zaha Hadid was born in Baghdad, Iraq in 1950, and earned a bachelor's degree in mathematics from the American University in Beirut, Lebanon, before moving to London to study architecture. In 1979, she founded her own company, Zaha Hadid Architects.

Among her most famous designs is the London Aquatic Centre, built for the 2012 Olympic Games and resembling a wave. Her style is often described as futuristic, with bold geometrical shapes, diagonal lines, and curves. This use of curves led many to think of her designs as a woman's response to living in a man's world and working in a field dominated by men.

She wanted to be respected for her work as an architect, not specifically a woman architect or a Muslim architect, but she hoped seeing her achievements would inspire young people from less represented groups to continue to break down barriers.

She holds many firsts as a woman in architecture, a field historically dominated by men. She was the first woman to receive the Royal Institute of British Architects (RIBA) Gold Medal and the Pritzker Architecture Prize. She also designed the Lois & Rosenthal Center for Contemporary Art in Cincinnati, Ohio, which was the first American museum designed by a woman.

Diébédo Francis Kéré

"Community was your family. Everyone took care of you and the entire village was your playground."

Francis Kéré was born in 1965 in Gando, Burkina Faso, a village that did not have a school. He left his family at the age of seven to attend school in another village with 100 classmates in a dark, hot building made of cement. He decided as a child that he would find a way to build better schools.

At 20, he moved to Berlin, Germany, to work as a carpenter while attending secondary school and university. While still a student, he started a foundation to create schools in Gando. The first, Gando Primary School (2001), was conceived, designed, and built with the help of the people in his home village. They worked with Kéré on every step of the process, used local materials, and built the school by hand.

Kéré graduated in 2004 with his degree in architecture. The Gando Primary School project earned awards and recognition, so he was able to establish Kéré Architecture in Germany. He continued to return to Gando to work with his hometown community to build more schools.

He also worked in Kenya, Mozambique, and Uganda to design schools and medical facilities, and has since built temporary and permanent designs in Denmark, Germany, Italy, Switzerland, the United Kingdom, and the United States.

Most importantly, in all of his projects, he engages the community throughout the process. He brings his technical expertise, but he listens to the needs of the people who live in the area and who will be using the spaces he creates. The community members are full partners in the design and construction process.

Maya Lin

"Keep a strong, clear vision."

Maya Lin grew up in Athens, Ohio, where her parents were on the faculty of Ohio University. Her father, Henry Lin, was dean of the art school and a ceramic artist. Her mother, Julia Lin, was a poet and professor of Asian and English literature. Both immigrated to the United States from China.

Lin displayed an interest and talent for mathematics and art as a young student. After high school she was accepted to Yale University. At Yale, she had to choose between architecture and art. She enrolled in the architecture program but still took sculpture classes.

Lin became one of the most controversial architect-artists in the United States when she was only 21. She designed the Vietnam Veterans Memorial in Washington, D.C., while still an architecture student at Yale.

The Vietnam Veterans Memorial Fund announced in 1980 that it would sponsor a nation-wide competition to design a memorial honoring those who had served in the Vietnam War. Nearly 1,500 aspiring artists submitted proposals. A panel of distinguished judges, including architects, sculptors, and landscape architects, chose the final design. Maya Lin's design is a simple V-shaped wall of polished black stone inscribed with the names of the 58,000 men and women who were killed in the war or declared missing in action.

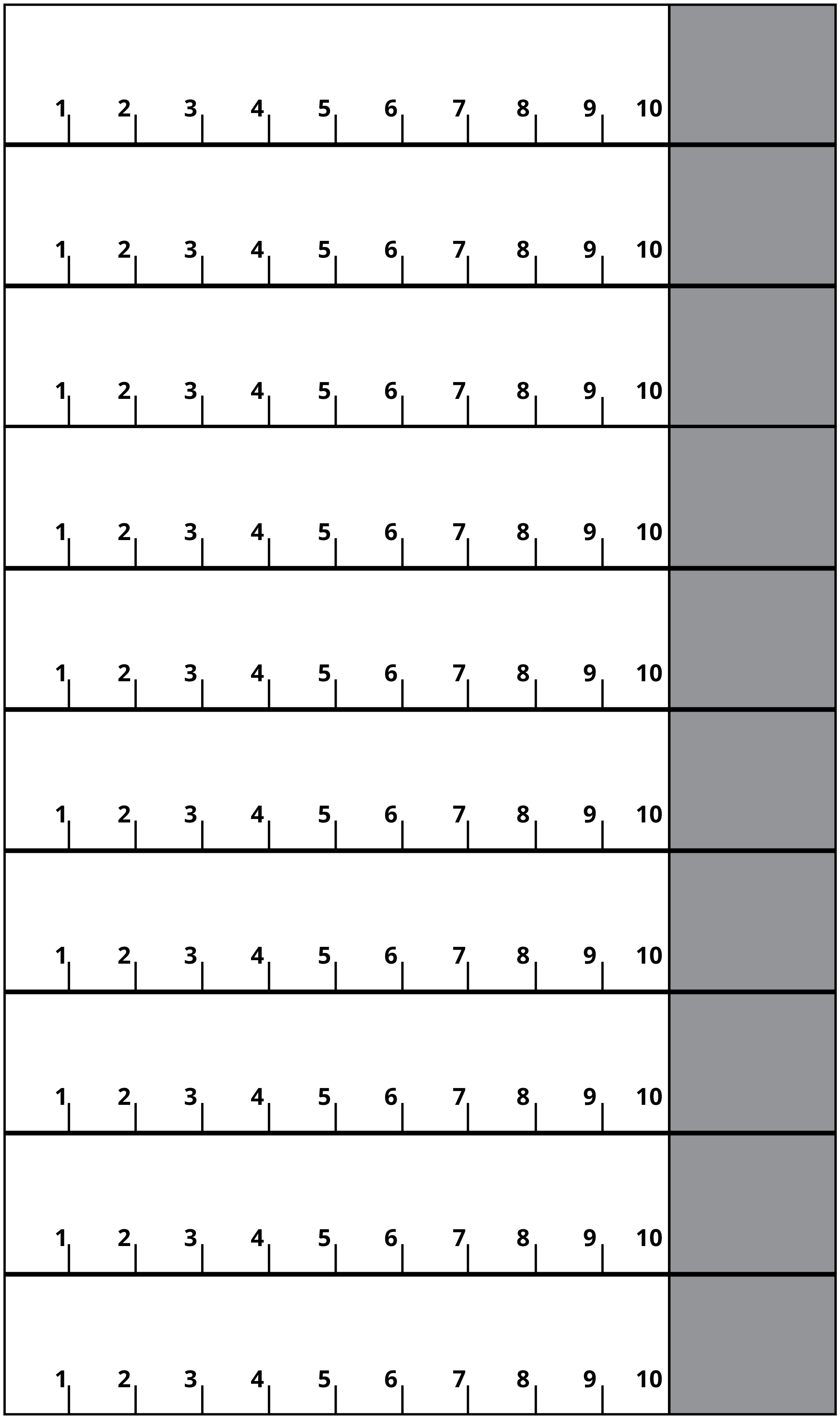
The design was very different from traditional memorials, and many people were angry at this less traditional design. However, since its installation, it has become the most visited memorial in the United States.

Lin has combined her interest and talent in art and architecture to create many landmarks since then. For her next project, a memorial for the civil rights movement in Montgomery, Alabama, Lin took her inspiration for the work from a quote from the Bible that Dr. Martin Luther King, Jr. used. He said that seekers of equality would not be satisfied until "justice rolls down like waters and righteousness like a mighty stream." The main sculptures in the memorial are a disk and a granite wall with water running over them.

Lin also feels strongly about environmental conservation, social justice, education, and the arts. To support these ideas she serves on the boards of The Southern Poverty Law Center's Teaching Tolerance project, the Kennedy Museum of Art, and Studio in a School in New York City.

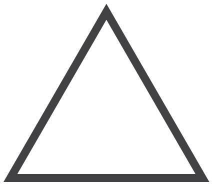
Read more about Lin at her studio's website, Maya Lin Studio.

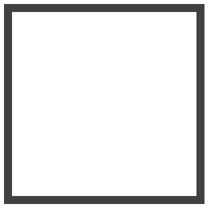
Metric Measuring Tape Template



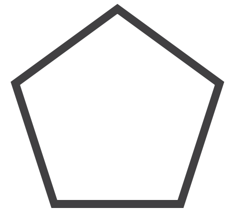
CENTIMETERS

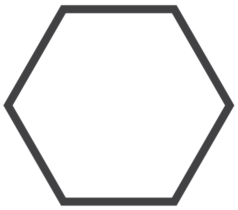
Geometric Shapes

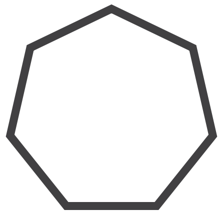

Triangle

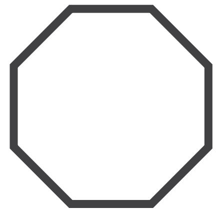

Square

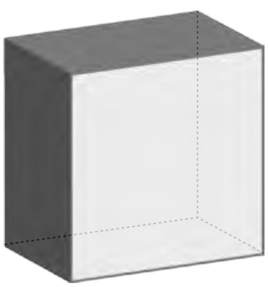

Rectangle

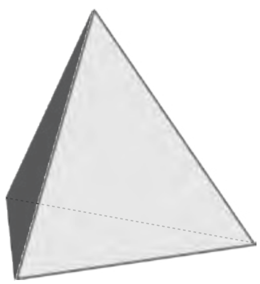

Pentagon

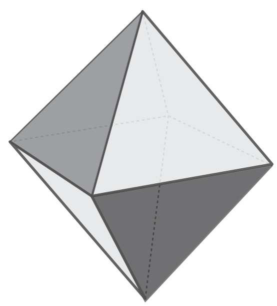

Hexagon

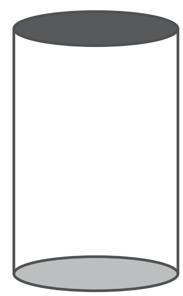

Heptagon

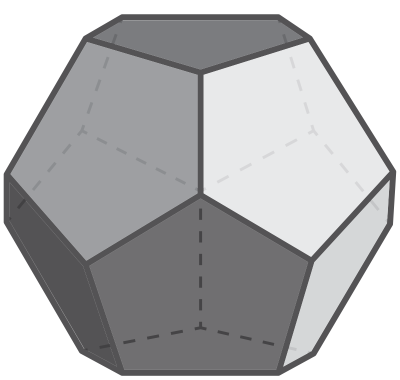

Octagon

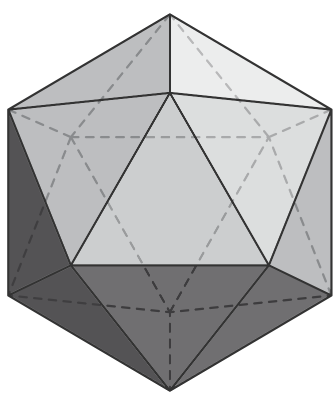

Cube

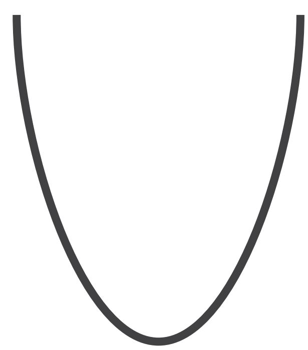

Tetrahedron


Octahedron


Cylinder


Dodecahedron


Icosahedron


Parabola

Design Questions

1. In what type of environment is your ideal community located? Is it in the country? In a city? Near a beach? In a suburb?

2. What natural resources are part of the environment? Water? Trees? Mountains?

3. Are there services that you would like to include in your ideal community that do not exist in your current community?

4. How do people get around in your community?

5. Where do people live in your community?

6. Are there open spaces set aside in your ideal community?

7. Are there issues that deal with accessibility that need to be addressed?

8. Are there facilities for older citizens?

9. Are young children and their needs considered in your ideal community?

10. Are there places for recreation and exercise in your ideal community? Playgrounds, pools, hiking trails?

11. Do people feel safe in your ideal community? Why?

Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

+8 Resources

Books

Adventures in Architecture for Kids: 30 Design Projects for STEAM Discovery and Learning, Vol. 2 (Design Genius, Jr., 2) by Vicky Chan (Beverly, MA: Rockport Publishers, 2021). Hands-on projects for kids to learn math, engineering, history, social studies, planning, geography, art and design skills through building challenges.

Architecture for Kids: Skill-Building Activities for Future Architects by Mark and Siena Moreno (New York: Rockridge Press, 2021). Hands-on activities introduce students to the basic terms and skills of architecture, examples of different kinds of buildings, and the ways architects look at the world.

Architecture for Teens: A Beginner's Book for Aspiring Architects by Danielle Willkens (New York: Rockridge Press, 2021). An overview of the basic elements of architecture, architectural movements and designers from prehistory to modern-day, and interviews with working architects.

Beijing: A Symmetrical City by Dawu Yu, adapted by Yan Liu, and translated by Crystal Tai (Richmond, CA: 1 Plus Books, 2020). Highlights the symmetry in architecture of Beijing as an introduction to Chinese history and culture.

Building Liberty: A Statue is Born by Serge Hochain (Washington, DC: National Geographic Society, 2004). Details about the creation of the Statue of Liberty from design to installation.

Cathedral: The Story of Its Construction by David Macaulay (Boston: Houghton Mifflin Company, 2010). Clear and fascinating commentary and detailed drawings show how a 13th century Gothic cathedral is constructed. A Caldecott honor book.

City: A Story of Roman Planning and Construction by David Macaulay (Boston: Houghton Mifflin Company, 1983). Detailed drawings and descriptions show how the Romans built exciting and functional cities for their citizens.

Dream Builder by Kelly Starling Lyons and Laura Freeman (New York: Lee & Low Books, 2020). The story of architect Philip Freelon, a black man whose father attended the 1963 March on Washington and who grew up to lead the team who designed the Smithsonian National Museum of African American History and Culture.

How Was That Built? The Stories Behind Awesome Structures by Roma Agrawal and Katie Hickey (New York, 2022). Explains the methods architects and engineers use to build some of the most unique structures in the world.

The Librarian Who Measured the Earth, Vol. 1 by Karen Lasky (New York: Little, Brown & Company, 1994). Tells the story of Eratosthenes, a librarian in ancient Greece, who figured out a way to measure the circumference of the earth using mathematics.

Magnitude: The Scale of the Universe by Kimberly Arcand and Megan Watzke (New York: Black Dog & Leventhal, 2017). A fully illustrated exploration and visualization of the concept of scale in the universe.

Mathematicians Are People Too: Stories from the Lives of Great Mathematicians, Vol. 1 by Luetta Reimer (Los Altos, CA: Dale Seymour Publishing, 1999). Stories that focus on moments of mathematical discoveries made by famous mathematicians.

Maya Lin: Artist-Architect of Light and Lines by Jeanne Walker Harvey and Dow Phumiruk (New York: Henry Holt and Co. Books for Young Readers, 2017). Tells Lin's story from a child observing the nature around her to designing the Vietnam Memorial and her adult life as an architect and artist.

Maya Lin: Thinking with Her Hands by Susan Goldman Rubin (San Francisco: Chronicle Books, 2017). An accessible biography for older or more advanced readers.

Measuring Penny by Loreen Leedy (New York: Henry Holt & Company, Inc., 2000). In this story Lisa measures her dog Penny, using all kinds of found objects.

Powers of Ten: A Flipbook by Charles and Ray Eames, presents photos of each level along with what you are looking at, one picture per page. (New York: W.H. Freeman Company, 1998).

Powers of Ten: About the Relative Size of Things in the Universe by Phillip and Phyllis Morrison (New York: Scientific American Library, distributed by W.H. Freeman Company, 1984). Each page moves inward, reduced at one-tenth the scale of the previous page. Available free through the Internet Archive.

Pyramid by David Macaulay (Boston: Clarion Books, Illustrated edition, 1982). Shows in detail how burial places for the rulers of ancient Egypt were conceived and constructed.

Why Humans Build Up: The Rise of Towers, Temples, and Skyscrapers by Gregor Craigie and Kathleen Fu (Victoria, BC: Orca Book Publishers, 2022). Asks why and how people build higher and higher buildings and what it means for the planet and environment.

The World Is Not a Rectangle: A Portrait of Architect Zaha Hadid by Jeanette Winter (San Diego: Beach Lane Books, 2017). Tells the story of Hadid growing up in Baghdad, Iraq, dreaming of building cities, and succeeding in her architecture career despite obstacles.

Zaha Hadid (Volume 31 of Little People, BIG DREAMS) by Maria Isabel Sanchez Vegara and Asun Amar (London: Frances Lincoln Children's Books, 2019). Part of a biography series for kids, this book tells Hadid's story from her childhood in Iraq through illustrations and historical photos.

Websites

15 Important African American Architects
A ThoughtCo. article listing brief profiles of black architects and their achievements.

AMS Equity, Diversity, and Inclusion Resources
The American Mathematical Society's page of resources supporting people interested in mathematical sciences regardless of gender identity or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, veteran status, immigration status, or any other social or physical component of their identity.

Architecture For All: 10 Thoughtfully Designed Buildings for People With Disabilities
This article from Architizer, written by James Bartolacci, highlights structures built around the world using principles of universal design for people with disabilities.

archKIDecture
ArchKidecture is an independent architecture education project that encourages children to explore and participate in the built environment. See list of books, links to biographies and activities for children.

Black Built Environment: Race and Architecture in America
A LibGuide from Pratt Institute Libraries compiling resources on diversity in architecture.

Design Squad
A PBS reality show where kids compete in engineering projects. The show targets after-school programs. Visit the website for information on past episodes, activities for youth, viewing schedule, downloadable educator's guide and newsletter.

Diversity in the Profession of Architecture
A report from the American Institute of Architects (AIA), 2016, of the results of a survey and study of who entered the profession, who prospered, and why in the previous ten years.

Ignite Worldwide
This nonprofit organization works to connect girls, young women, nonbinary, transgender, agender, and genderqueer students with STEM opportunities during the school day. The site features several stories and features that students may be interested in, including IGNITE Panels, videos of conversations with women in STEM careers.

Increasing Diversity in Architecture
From Architecture magazine in 2019, a three-part series on how to make architecture more inclusive and representative focusing on barriers to entry, designer-led solutions, and firm initiatives.

Math Glossary: Mathematics Terms and Definitions
This site hosts an expansive glossary of math terms and links to math tutorials in subjects like geometry.

Society of Women Engineers (SWE)
This site features role models, scholarship information, and statistics on the participation of women in the field, diversity initiatives, and resources for educators.

The STEM Gap: Women and Girls in Science, Technology, Engineering and Mathematics
This American Association of University Women (AAUW) site compiles the data on people identifying or presenting as women studying and working in STEM fields and provides research-based programs and solutions to people identifying or presenting as women being underrepresented in these fields.

Very few architects are Black. This woman is pushing t change that
A National Public Radio (NPR) interview profile of Pascale Sablan, serving as president of NOMA, the National Organization of Minority Architects, working for a prestigious firm, and the youngest African American inductee of the AIA College of Fellows.

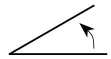
Afterschool Math Plus

Revised Edition

Theme +2: Built Environment

+9 Glossary and Materials List

Glossary

Acute angle An angle of less than 90° 

Cartography The art or technique of making maps or charts

Centimeter 1/100 of a meter

Charrette The term "charrette" originally meant an architectural student's final work, representing what they had learned. It has come to mean an intense period of design activity in a group, especially a group that includes representatives of the local community who live or work in the area being designed, to meet a deadline in fields like architecture, landscape architecture, industrial design, interior design, graphic design, etc.

Compass An instrument (sometimes called a bow compass) with two adjustable "legs" for drawing circles and line segments

Data Facts or numbers that describe something

Inventory The process of making a list, report, or record

Legend Table or list of the symbols appearing on a map or chart

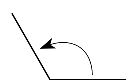
Map Key A reference point that gives information on symbols or scale in a map; also see legend

Measurement The geometric function that describes the distances between pairs of points in a space

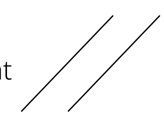
Meter An international standard unit of length, approximately equivalent to 39.37 inches. It was redefined in 1983 as the distance traveled by light in a vacuum in 1/299,792,458 of a second.

Metric system A system of measuring lengths, volumes, and weights, using units such as meter (for length), kilogram (for weight), and liter (for volume)

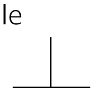
Nonstandard units of measurement Units of measurement that are not commonly accepted: a made-up unit

Obtuse angle An angle greater than 90° and less than 180° 

Parabola The U-shaped curve that is the graph of a second-degree equation

Parallel Two or more straight lines in the same plane that do not intersect but are an equal distance apart at every point 

Patterns A set of objects that is generated by following a specific rule

Perpendicular Intersecting at or forming right angles (90° angle) 

Proportion An equation of fractions in the form a/b=c/d

Protractor An instrument for laying down and measuring angles in drawing or plotting.

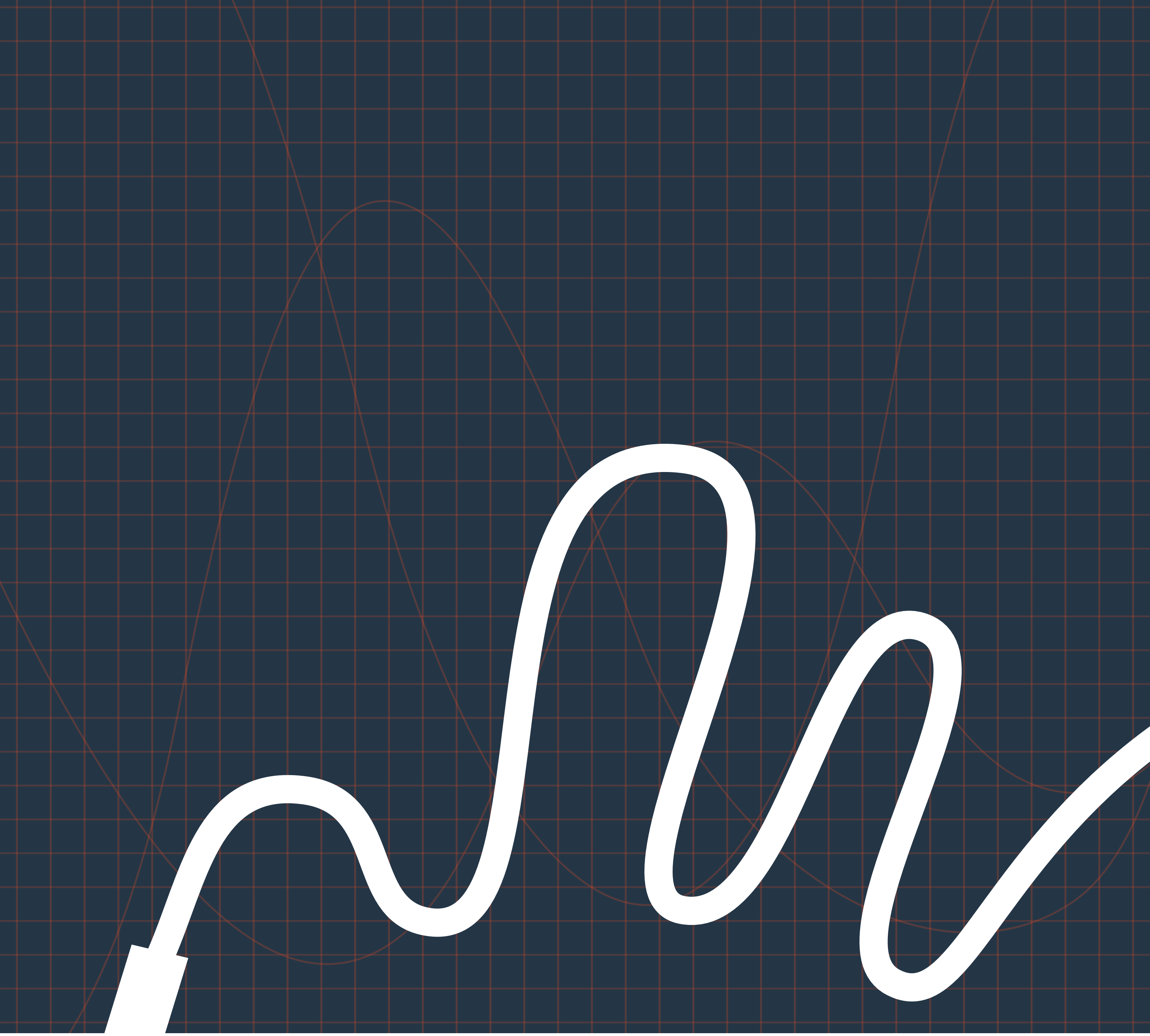
Ratio A comparison of two quantities that have the same unit of measure

Scale The relationship between distances on a map and the distance in the real environment

Standard units of measurement Commonly accepted units of measurement

Materials List

- ☐ Centimeter graph paper
- ☐ Chart paper
- ☐ Chart-size centimeter graph paper
- ☐ Clear tape
- ☐ Clips to make clipboards
- ☐ Clothesline rope
- ☐ Colored masking tape
- ☐ Compass
- ☐ Crayons
- ☐ Data sheet
- ☐ Drawing paper
- ☐ Drawings and maps from previous activities
- ☐ Light cardboard
- ☐ Magnifying glasses
- ☐ Markers
- ☐ Masking tape in two colors
- ☐ Metric measuring tapes
- ☐ Paper
- ☐ Pencils
- ☐ Pieces of cardboard
- ☐ Pieces of hook-and-loop tape
- ☐ Protractor
- ☐ Rough sketch of the ideal community (from Activity 4)
- ☐ Selection of maps
- ☐ Tape
- ☐ Wax-covered yarn



Afterschool Math Plus

Theme +3: Jump Rope Math

fhi360

Introduction

Did you ever think that students could learn essential math skills while jumping rope, having fun, and getting exercise? Well, that is what this theme is all about. The activities center on jump ropes—familiar objects we all have used on the playground, at the gym, or for serious athletic training—as vehicles for collecting, representing, and interpreting data.

As they work on the activities, students create bar, line, and scatter graphs, and Venn diagrams; they conduct and analyze surveys; they measure various jump ropes in relation to their own heights using standard and nonstandard measuring tools; and they count by multiples as they plot information on their graphs. By collecting and reflecting on data, students develop understanding and skills that are essential for informed decision making.

Equity

The Jump Rope theme is a good introduction to equity. Jumping rope requires coordination, strength, and skill to do it well. Through these activities, students will learn about the value of jumping rope for building stamina, aerobic strength, and coordination, and at the same time they are learning math! Jumping rope isn't an activity for a certain group of students, it's for all students.

Inclusion

There are several adaptations that should be considered to promote inclusion of students with disabilities. For example, for students who are visually impaired, using a brightly colored rope or painting the ends a bright color

NCTM Math Standards

Content Standards

- **Data Analysis and Probability:** Formulate questions that can be addressed with data, and collect, organize, and display relevant data to answer them.
- **Numbers and Operations:** Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; and apply and adapt a variety of appropriate strategies to solve problems.
- **Communication:** Organize and consolidate mathematical thinking through communication; communicate mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely.
- **Representation:** Create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena.

will increase visibility. Since sound is produced when the rope hits the ground, allow students who are visually impaired to listen to several rope turns in order to get the rhythm before jumping in. Students with and without disabilities can be paired as jumpers. This will not only promote inclusion, but teamwork and communication skills as well.

For students with physical or developmental disabilities, the rules of the game can be adapted. Students turning the rope can stop when the rope reaches the lowest point and count or spell. This will allow additional time for a student with a disability to jump or cross over the rope. It will also be great practice for the turners to increase their spelling and math skills. Using this method allows students with and without disabilities to participate at their own level. Specific modifications for students who use wheelchairs are discussed in the activities.

Since some students are oral learners and some are visual learners, give students the activity instructions both ways. As you are telling students the instructions, chart them. This will also help students who may need to refer to the instructions again.

Remember that students with disabilities are already problem solvers and will have figured out their own way to accomplish tasks based on their strengths. Be open to all approaches to success!

Cultural Links

Jump rope games can be traced back to 1600 B.C. when the Egyptians used vines for jumping. Now it has become an official sport with international competition. Be sure to encourage students to share various jump rope games—e.g., double dutch, Chinese jump rope—from their own cultures (see the Resources Section for websites about the history of jump rope games).

Literacy and Math Identity

Three young women from diverse backgrounds who think math is exciting and use it every day will inspire students to think about math in new ways: Monica Barnes, an electronic engineer; Tahani Amer, an aerospace engineer; and Melanie Wood, the first woman to win the U.S. Mathematics Olympiad. Additionally, Bernard Morin, who became blind at age six, became a well-known mathematician with a specialty in topology. Be sure to read their bios and make time for students to do the career and role-model activities found at the end of the theme.

At the Museum

After completing the series of Jump Rope activities, students work on creating exhibits at the partnering museum. Museum staff, often teen "explainers," help students design exhibits based on the activities and the investigations they have planned and carried out at their afterschool center. Museum staff also work with students on their presentation skills, giving pointers about voice projection, body language, and other skills.

Since the math in this theme is about collecting and interpreting data, student exhibits take the form of different kinds of graphs—bar, line, scatter, and Venn diagrams. Students often add a hands-on element to their exhibits, inviting family members to try jumping rope in a variety of ways.

It was difficult for some of the boys to learn how to jump rope since they had never done it before. In the end, it opened up a whole new world for them. – Center Director

At the Jump Rope Math culminating event, students presented exhibits that explained how they had collected data on various aspects of jumping rope. Then, they asked family members to participate in similar experiments, and plotted the data on graphs right then and there. People of all ages jumping rope added even more fun to the event!

Remember to:

- ☐ **Check out the resources.**
- ☐ **Review the glossary.**
- ☐ **Send out the family letter when you start the theme.**

Dear Families,

Our current theme in the Afterschool Math Plus program is Jump Rope Math! Each week your child will participate in a hands-on math activity that will reveal and reinforce math skills and concepts as well as make connections to mathematics in the world around us. Your child will have fun jumping rope (or, learning to jump rope), while exploring the various materials used for making jump ropes, experimenting with, measuring and comparing various jump ropes, and collecting, displaying, and analyzing data on different types of graphs. You didn't think there was much math in using jump ropes? But there is!

The goal of this program is to widen the possibilities for your child's future, and in addition to finding the math in everyday things, your child will learn about career options and diverse math role models whose stories are encouraging and engaging, and who attribute much of their success to the encouragement of the adults in their lives.

There are many ways you can support your child's success with Afterschool Math Plus. Join in the fun. Share your own jump rope stories with your child. Ask your child to tell you about Jump Rope Math activities. Visit a local science museum to explore the exhibits and talk about the math in everyday things.

If you have access to a computer, visit the Internet for information on the Double Dutch Divas, women with lots of energy and good timing and/or the International Jump Rope Union for pictures of jump rope skippers from around the world. Be sure to ask your child about the careers and role models highlighted in the program and encourage them to see mathematics as a fun and possible career option.

Please feel free to ask a staff person about Afterschool Math Plus and your child's participation.

Sincerely,

Queridas Familias,

¡Nuestro tema actual en el programa las Matemáticas Despues-de-Escuela y MÁS es Jump Rope Math (Matemáticas de Salto de Cuerda)! Cada semana su niña/niño participará en una actividad de las matemáticas en que se usan las manos, que revelen y refuercen las habilidades y conceptos de las matemáticas y las conexiones a las matemáticas en el mundo alrededor de nosotros. Su hijo se divertirá saltando la cuerda (o, aprendiendo a saltar la cuerda), mientras que explora los varios materiales utilizados para hacer cuerdas para saltar, y experimentar, medir y comparar varias cuerdas para saltar, y a recoger, exhibir, y analizar datos en diversos tipos de gráficas. ¿No pensó que había muchas matemáticas al usar la cuerda para saltar, verdad? ¡Pero las hay!

La meta de este programa es ensanchar las posibilidades del futuro de su niña/niño, y además de encontrar las matemáticas en cosas diarias, su niña/niño aprenderá sobre las opciones de carreras y los modelos diversos de la matemáticas e historias que animan y que atribuyen mucho de su éxito al estímulo de los adultos en sus vidas.

Hay muchas maneras que usted puede apoyar el éxito de su niño/niña con matemáticas de la Matemáticas Despues-de-Escuela y MÁS. Participe en la diversión. Comparta su historias de saltar la cuerda. Pida que su niña/niño le diga sobre actividades de Jump Rope Math (Matemáticas de Salto de Cuerda). Visite un museo local de la ciencia para explorar los objetos expuestos y para hablar de las matemáticas en cosas diarias.

Si usted tiene acceso a una computadora, visite el Internet para la información sobre las divas del "double-dutch", mujeres con mucha energía y buena sincronización, y/o la Unión Internacional para Saltar la Cuerda para ver retratos de saltadores de cuerda de todo el mundo. Asegurese preguntarle a su niña/niño acerca de las carreras y de los modelos positivos destacados en el programa, y anímelo a que ella/el vea las matemáticas como una diversión y posible opción de carrera.

Por favor, sientase cómodo/a de preguntarle a nuestros empleados sobre el programa Matemáticas Despues-de-Escuela y MÁS y de la participación de su niña/niño.

Sinceramente,

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

Who Uses Math? Equity Activity +3: Using Math Every Day

Who Uses Math?

Using Math Every Day

It is important for all students to know that math is all around us every day, that we all use math every day, and that math is a basic skill that everyone needs to be successful. Be sure to help all students feel comfortable about doing math and be open to the excitement and elegance that math offers!

1. Ask students to think about when they use math—playing a game and keeping score, baking cookies and measuring the ingredients, drawing a picture with perspective, learning to read music, measuring how tall they are, and thinking about the win records of tennis players.

2. As a group, make a list of "math uses" that includes these ideas and others.

3. In small groups, ask students to draw pictures of people doing math. Display the pictures and ask students to talk about what they included.

Note to Group Leaders:
It is important to consider who students included in their drawings. Are there any patterns in the drawings related to:

- Age
- Gender
- Ability
- Diverse backgrounds

Jessica Scranton for FHI 360.

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

+1 Exploring Jump Ropes

+1 Exploring Jump Ropes

ASM+
INSTRUCTOR GUIDE

Question

What materials are used in making jump ropes? How can we use a jump rope for exercise and fun?

Objectives

Students will:

- Demonstrate their methods for jumping rope.
- Have an opportunity to try each other's ideas for jumping rope.
- Investigate a variety of materials used in making jump ropes.
- Represent data collected in small groups using a bar-graph format.
- Share their findings with other groups.
- Combine information to generate a bar graph that includes data for the entire group.

Where's the Math?

The saying, "One picture is worth a thousand words," applies to math, too! Graphing is a visual way to represent data. Or to say it another way, graphs are pictures that tell us about numbers. The graphs in this activity will represent data that students collect about the materials used in making jump ropes and the various jump-rope techniques students know. By reading graphs students will be able to compare totals and draw conclusions.

NCTM Math Standards

Content Standards

- **Data Analysis and Probability:** Formulate questions that can be addressed with data, and collect, organize, and display relevant data to answer them.

Process Standards

- **Communication:** Organize and consolidate mathematical thinking through communication.
- **Representation:** Create and use representations to organize, record, and communicate mathematical ideas.

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Math Skills Developed

- Collecting data
- Representing data
- Constructing a bar graph
- Interpreting a bar graph

Materials

- ☐ Jump ropes brought from home
- ☐ Extra ropes made from a variety of materials
- ☐ Sticky notes
- ☐ Pencils
- ☐ Chart paper
- ☐ Markers
- ☐ Bags (recycled plastic or paper)

Handouts

- ☐ Small-Group Investigation Bar-Graph Template—enough copies for each small group and one chart-sized enlargement
- ☐ Large-Group Investigation Bar-Graph Template

Getting Ready

A few days before you plan to do this activity, ask each student to bring in a jump rope from home, if they have one. As students begin to bring in their jump ropes, put each one into a bag clearly labeled with the student's name. Collect additional jump ropes to ensure that there will be a variety of materials (cotton, plastic, rubberized plastic, plastic links, elastic, etc.) as well as enough jump ropes for all. You can also cut lengths of standard rope; this may be the most economical and equitable method. If possible, conduct the activity in a large classroom, the cafeteria, outside, or the gym. Use the template to prepare a chart-sized group bar graph; across the bottom (on the horizontal or x axis) record the various kinds of jump rope materials you have collected.

Introduction (10 minutes)

Bring students together. Hand out the jump ropes and supply ropes to students who have not brought one from home. Mention that many of us know people who use jump ropes for exercise and fun. Tell students that during this activity, they will share some of the things they know about jump ropes (including games and techniques) and have fun while doing it!

Part One: Large-Group Graphing (10 minutes)

1. Display the large bar-graph template you have prepared. Have sticky notes and pencils ready. Ask students if they have ever made a bar graph and to describe any bar graphs they have made.

> *Note to Group Leaders:*
> It's possible that even though students have had experience with bar graphs (birthday months, favorite foods, etc.), they may not be familiar with the term.

2. Ask students to observe the ropes. Are all the ropes the same? Different? If students are not sure, help them to identify the materials used in their jump ropes (cotton, plastic, rubberized plastic, plastic links, elastic, etc.).
3. Have students write their names on a sticky note and place it on the bar graph above the material that corresponds with their rope. Make sure that the bar graph is placed at a height accessible for all students, particularly those using a wheelchair or other adaptive equipment.
4. Discuss what the bar graph shows. What material was most represented in the ropes? Least represented? Any idea why? How many more students had a rope made of plastic rather than plastic links? Did anyone have an elastic rope? Did all the ropes have handles?
5. Once students have analyzed the data, explain that they will be working in small groups to create their own bar graphs about various jump-rope games and techniques.

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Part Two: Small-Group Investigation (20 minutes)

1. Organize students into small groups of four to six. Assign each group to a section of the available space. Make sure that each small group has a variety of ropes and enough room to jump.
2. Ask students to take turns demonstrating one way to use a jump rope for exercise or fun. Remind students that each person needs to have at least one turn to demonstrate a technique. Circulate to ensure that all groups are working at achieving this goal.

- Students will want to jump and have fun. Give them a little time at the beginning to show off their skills and creativity.. Be sure that students understand the need to give each jumper enough space so that the rope will not accidentally hit other students.

- Encourage all attempts at participation and maintain an open mind about the different ways children might be able to jump. Some students with motor disabilities might prefer to place the jump rope on the ground and jump over it. A student who uses a wheelchair might wish to have the ropes create a path to challenge them to follow.

3. After everyone in the group has had a turn, ask students to try out each other's methods with different kinds of ropes.

4. Distribute a bar-graph template (with grid) to each group. The horizontal (x) axis is identified as the different jump rope techniques and the vertical (y) axis the number of students.

5. Ask students to each write their name in one block on the graph indicating their jump-rope technique (for example, snake, criss-cross, fast skipping, one foot at a time). Once they have finished constructing their small-group graph, ask students to put their ropes away in a space you have designated and gather everyone.

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Part Three: Sharing Findings (15 minutes)

When everyone has gathered, ask students what they learned about how people jump rope. Encourage everyone to share in the discussion.

Show students the large bar graph they made at the beginning of the activity. Ask them to compare it with the graph they created in their small groups. How are the graphs similar? Different?

Tell students that you want to transfer the data that they collected in the small groups to a large graph. Look at the first category. If the first category is "fast skipping" ask students, "How many had fast skipping as a category?" Ask each group to report how many students used that technique and have a student from each group come up and color in the number of corresponding blocks on the large graph. Again, make sure that the bar graph is at an accessible height for all students. If a student has fine-motor challenges that prevent use of the marker, have paper squares with tape on the back or sticky notes for the student to place on the graph. If students who are colorblind are present, consider using color and shape when marking blocks. For example, make red Xs in one column, blue check marks in another.

Use this method to move through each category. Once each category is completed, ask if there are any other jump-rope techniques that a group wants to add to the graph, either as a new category or under a general category labeled "other."

When the group bar graph is completed, ask students if they can draw conclusions based on the graph. Some questions to ask are: "Which technique is used by most students?" "Which one the least?" "Did anyone use the snake?"

Encourage students to draw as many conclusions as they can by observing the information on the graph. Record their conclusions on a piece of chart paper to display with the graph.

Let students know that there will be more jump-rope activities coming up. Post a chart labeled, “Our questions about jumping rope.” Encourage students to write down questions they have as they do the activities and tell them that they will be able to find answers to the questions later on. At the beginning of each activity, start the chart by asking, “Does anyone have a question to post today?”

Part 4: Literacy and Math Identity Activity (60 minutes)

1. Make a parabola book. Ask students to think about the shape the jump rope makes as it turns. Students may describe the shape as an arc or a bridge. Explain that the mathematical name for this shape is a parabola—a mathematical curve that is important in geometry.

2. Ask if anyone can think of something that has this geometric shape. If students need help, you might ask them to think about the shape of a spout of water in a water fountain, a suspension bridge (e.g., the Golden Gate Bridge in San Francisco), or the arc of a baseball when thrown.

3. Have students look up parabola in the dictionary or online.

    - Have students read “How (and why) do NASA researchers simulate microgravity on Earth?” and “Katherine Johnson: Pioneering NASA Mathematician,” by Emily Staniforth and Joanna Stauss. Ask them to look for information on parabolas in these (and other space-related articles they may find online) and how knowing about parabolas contributed to space flight. Remind them that the shape they’re reading about in regard to space flight is the same as the one the jump rope makes as they jump.

4. Collect some magazines (science and travel magazines work well) and have students cut out pictures of parabolas (fountains, bridges) and make a book of parabolas. Students can also add their own drawings of parabolas, or images printed from the Internet. Use this opportunity to show them how to cite the sources they used to find the images.

5. Have older students explore the geometric properties of parabolas and write captions for the images to explain these properties.

6. Display the book(s) at the center, or publish on the center’s website.

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Additional Literacy and Math Identity Activities

If one of these activities better suits your students or the goals of your afterschool program, replace Part Four with it. Or add these activities into the math to provide extra math-identity building.

- Use the data from the bar graphs to create a jump-rope rhyme.
  - For example: Three ropes were cotton, ah, ah, ah
    Two were elastic, ha, ha, ha
    One was plastic, pa, pa, pa
    Now I'm leaving, ta, ta, ta
- Ask students to name the jump roping rhymes and games that they know or have played before. Make a list on chart paper. Have students choose which ones they want to learn to play and focus on for the literacy activity. Have students who are experts in those rhymes or games teach them to everyone. Split the students into smaller groups and have each group write the instructions for how to learn the rhyme or the game.
- Have students read online about Tiera Guinn Fletcher, an engineer at Boeing who worked on NASA's Space Launch System that will send astronauts and cargo to the moon. She was also in a jump rope club when she was growing up in Georgia. Ask them how finding mentors made a difference in her decision to pursue science and engineering in her professional life. Possible articles include:
  - "Wonder Woman of Rocket Science," by Stephanie M. McPherson on *Slice of MIT*, the MIT Alumni magazine online (May 18, 2022).
  - "Before working on spacecraft, this engineer overcame self-doubt," by Carolyn Wilke on *Science News Explores* online (April 7, 2020).

Equity

- Did all the students have an opportunity to jump rope?
- Did students with disabilities actively participate? Were accommodations made as needed?
- Were students supportive of those who did not know how to jump rope?

Reflecting on the Activity

- Was I able to encourage students who had not jumped rope before?
- Did students with disabilities participate in ways other than rope turners?
- Was I able to make the activity noncompetitive?
- Did students feel good about their jump-rope skills?
- Were students familiar with bar graphs?
- Is there additional information students need in order to understand a bar graph?

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

+2 The Whys of Jumping Rope

+2 The Whys of Jumping Rope

ASM+
INSTRUCTOR
GUIDE

Question
Why do people jump rope?

Objectives
Students will:

- Conduct a survey and share the results with the group.
- Develop possible categories for a group bar graph based on the data they have collected.
- Construct a group bar graph using multiples for an axis.
- Construct a Venn diagram to document their findings.
- Interpret information on the bar graph and Venn diagram.

NCTM Math Standards

Content Standards

- **Data Analysis and Probability:** Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

Process Standards

- **Connections:** Recognize and use connections among mathematical ideas.
- **Communication:** Organize and consolidate mathematical thinking through communication.
- **Representation**: Create and use representations to organize, record, and communicate mathematical ideas.

Where's the Math?
A survey is a way of collecting data. In this activity students will conduct a survey and create a bar graph and a Venn diagram to document their findings. The graph and diagram will help students to interpret the data and draw conclusions.

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Math Skills Developed
- Conducting a survey
- Reporting data
- Constructing a bar graph
- Formatting using multiples
- Counting by multiples
- Using a Venn diagram as a tool for organizing data
- Interpreting data

Materials
- ☐ Pencils
- ☐ Paper
- ☐ Chart paper
- ☐ Markers
- ☐ Ruler

Handouts:
- ☐ Sample Survey: The Whys of Jumping Rope
- ☐ Bar Graph Template
- ☐ Sample Bar Graph 1
- ☐ Sample Bar Graph 2

Getting Ready
A few days before you plan to do this activity, discuss the idea of a survey with students. Have they ever participated in a survey? Why are surveys used? What kind of information can we get from a survey? Explain that a survey is a way of collecting information about groups. Make enough copies of the sample survey so that each student will have at least three.

ASM+
ACTIVITY

Introduction

This activity uses a simple survey that will provide students with a broader view of the reasons people jump rope. Mathematically, it also helps students to understand that when dealing with larger numbers, you do not need a larger piece of paper. A visual aid, such as a graph, provides a "snapshot" of the data and is easier to understand. Visual representation of the data also helps with making predictions and demonstrates that there is more than one way to document the same data.

Part One: Developing a Survey (15 minutes)

1. Explain to students that they will be conducting a survey that will provide important information about the "whys" of jumping rope. The survey will focus on finding three people who have jumped rope. The people can be members of their extended family, friends, or neighbors.

2. Discuss the sample survey with students. Do they understand the question? Do they want to practice conducting the survey with each other? Ask students if they want to add any questions to the survey. Students will be graphing the answers to the question, "Why do you jump rope?" But they also might want to include questions about when the person started jumping rope, where they jump rope and their favorite techniques.

3. For students with learning disabilities, it might be helpful to provide a survey template (including respondent's name, why they jump rope, and other notes). For students who are visually impaired, use of a voice recorder on a mobile device or computer might be encouraged for survey interviews.

4. Remind students that they need to find three people to survey and write down the responses they get. Note that people answering may have more than one response. For example, someone may jump rope for exercise **and** fun. Agree on a deadline for when the surveys will be completed and brought back to the whole group.

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Part Two: Processing Data (20 minutes)

1. Organize students into small groups of four to six and ask them to discuss what they found out through their surveys. What answers did they get to their questions? Ask one person from each group to volunteer to collect responses to the question, "Why do you jump rope?" Remind them that there are many possible answers, and they should all be included.

2. Explain that students are going to make a graph of the information they collected. Remind students by showing them the model of the bar graph used in the first activity, "Exploring Jump Ropes."

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ASM+ ACTIVITY

3. Ask each group to look at the survey results. Show students the graph template and ask them to think about labels for the x and y axes (explain that the x axis is horizontal, like the horizon, and the y axis is vertical, like a ladder).

4. After labeling the graph, have students transfer the information (data) from the survey onto their group graph. Remind students that, when working in groups, everyone must be part of the discussion and everyone's idea counts.

5. If you have students with hearing impairments, consider use of the "pass the microphone" technique in this and all small-group activities. By designating a "microphone" (such as a pencil or paper cone) and establishing that only the person with the microphone can speak, you enable the student who is deaf or hard of hearing to have a visual cue of who is speaking so that they can lip read or simply focus on a single voice.

6. Explain that each group will present their graph to the large group. The presentation should include how the axes are labeled.

> *Note to Group Leaders:*
> As groups begin to work, check to see that each student understands the task. Does each group have an idea about how to start? Encourage students to come up with their own ideas. Assist groups that need support in completing the assignment. Remind groups a few minutes before the end that time is winding down.

Part Three: Sharing Group Findings (15 minutes)

Ask one group to volunteer to present its graph. Encourage other groups to listen to the presentation to see if they collected similar or different information. At the end of the presentation, ask if anyone has a question.

The next step in the process is to create a large-group graph based on the answers to the question, "Why do you jump rope?"

Part Four: Large-Group Graphing (20 minutes)

> *Note to Group Leaders:*
> One factor that makes this a challenging activity is the number of responses that need to be incorporated into the bar graph. If you have 20 students, and each student has conducted the survey three times, there are 60 potential responses. If everyone gave the same answer, then you would have one answer repeated 60 times. This is not likely, but it is an interesting possibility to discuss. This activity is structured to provide a variety of leadership roles for students. Be sure that students who don't usually volunteer have an opportunity to act as leaders.

1. Taking turns, list all the responses each group reports. Once all responses have been recorded on the board or chart paper, count the categories. Can some categories be combined?

2. Line up the categories along the x (horizontal) axis and use the y (vertical) axis to chart the numbers.

    - Point out to students that, given the large numbers of responses, it might be helpful to count by threes (or another multiple) for the number axis. Ask students to help you count along the axis by counting by the number they decide to use.

3. Create the group bar graph of their survey:

    - Select a particular response and ask the groups to indicate how many times they received that answer. For example, ask students how many people said they jumped rope for fun? Count the total number of responses. Be sure to count aloud to reinforce connection between responses and final numbers for students who have learning and/or visual impairments.

    - Ask students if they can tell you where the top of the bar needs to be to show this number. Check for agreement on this question before moving on. If students need to count by threes (or another multiple), do that once again to clarify how the correct answer can be attained. Ask a student to color the bar to the level needed to reflect the correct number of responses for that category.

    - Repeat the last three steps until all categories are represented on the bar graph. Use a different color and/or technique for each category to make the bar graph visually accessible for students who are colorblind or have low vision.

    - Ask students how they would answer the question, "Why do you jump rope?" Have students add their answers to the bar graph.

4. Once the bar graph is completed, ask students to decide on a title. Then ask them to develop questions that can be answered using the information in the graph. For example:

    - Do more people jump rope for fun or exercise?
    - Do people jump rope for more than one reason?
    - What is the total number of people represented?
    - Can the results be placed in rank order? Does it make a difference on the graph?
    - Which response had the lowest number of people? The most?

5. Write the questions on chart paper and post alongside the bar graph in a central location so that other students at the center can write their responses.

Part Five: Literacy and Math Identity Activity (20 minutes)

1. Have students write a survey to find out information about the afterschool center/program.

    - Questions could include general information, such as number of students, activities, students, and number of staff members, as well as information about favorite movies, favorite games, and favorite center activity.

2. When students have gathered their data, have them create bar graphs to represent it in a visually pleasing and easy-to-understand way. Have them create their graphs by hand using arts and crafts materials or using a digital tool like Excel or Google Sheets.

3. Have students write brief introductory paragraphs explaining their surveys and post the answers to the surveys and/or their findings graphs in the center's newsletter or website.

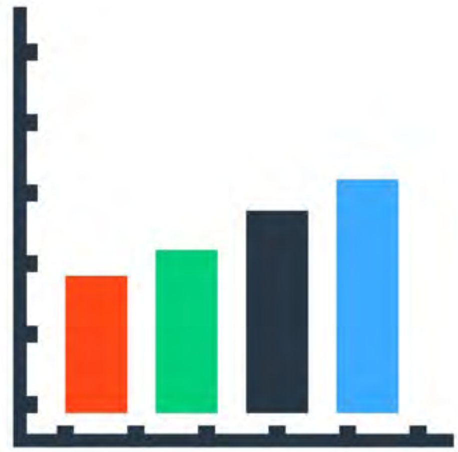
Part Six: Creating a Venn Diagram (10 minutes)

1. On chart paper, draw three intersecting circles. Label the circles: Exercise, Fun, Other. Explain that this particular diagram is called a Venn diagram. It allows us to put things in each circle that are unique to that circle, yet it

also allows us to put things that are common to these items in the overlapping area of the circles. Explain that students will be using the Venn diagram as another way to record the survey responses.

2. Based on the responses, ask students to fill in the circles of the Venn diagram by indicating the number of responses in each area of the diagram. Then have them compare the Venn diagram to the group bar graph. What does the bar graph tell them that the Venn diagram does not, and vice versa? Display the Venn diagram alongside the group bar graph.

Note to Group Leaders:
Remind students to write down any questions they have on the "Our Questions about Jumping Rope" chart.



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Additional Literacy and Math Identity Activity

If this activity better suits your students or the goals of your afterschool program, replace Part Five with it. Or add this activity into the math to provide extra math-identity building.

- Have students go to the U.S. Government Census Bureau website. Explain to students that the U.S. Census is a survey just like the one they conducted and that it is the largest survey ever conducted! Help students use the website to find out how many people live in their community, town, city, or state. Compare it to information about another town, city, or state.

Equity

This activity is structured to provide a variety of leadership roles. Be sure that students who don't usually volunteer have an opportunity to act as leaders.

Reflecting on the Activity

- Did everyone participate in collecting survey data?
- Did students work well together in creating the large bar graph?
- Were students of different gender identities equally involved?
- Were accommodations made as needed for students with disabilities?
- Did students understand how to interpret the information on the two different types of documentation?

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

+3 How Far Can You Go?

+3 How Far Can You Go?

ASM+
INSTRUCTOR GUIDE

Question
Can you predict how far you can skip rope in a given amount of time?

Objectives
Students will:

- Observe and measure jumping rope across a distance.
- Collect and record data.
- Construct a line graph using the data collected.
- Analyze line graphs to hypothesize what might happen with time as a variable.
- Develop hypotheses and share findings.

Where's the Math?
Students will construct a line graph that can be used to predict change in distance in a given amount of time. Using the data collected as coordinates, they will create a line graph to demonstrate their findings and to make predictions about change.

Math Skills Developed
- Collecting data
- Constructing a line graph
- Plotting coordinates
- Interpreting a line graph
- Developing hypotheses
- Checking hypotheses based on data

Materials
- ☐ Jump ropes
- ☐ Pencils
- ☐ Meter measuring tape
- ☐ Chart paper
- ☐ Markers
- ☐ Rulers
- ☐ Sheet of paper for recording data

Handouts
- ☐ Line-Graph Template A
- ☐ Sample Line Graph

NCTM Math Standards

Content Standards

- **Algebra:** Understand patterns, relations, and functions.
- **Number and Operations:** Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- **Measurement:** Understand measurable attributes of objects and the units, systems, and processes of measurement.
- **Data Analysis and Probability:** Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through problem solving; apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematical problem solving.
- **Representation**: Create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena.
- **Reasoning and Proof:** Recognize reasoning and proof as fundamental aspects of mathematics; make and investigate mathematical conjectures; develop and evaluate mathematical arguments and proofs; and select and use various types of reasoning and methods of proof.

Getting Ready
Provide a variety of jump ropes—enough for each pair of students to have one. Prepare a hypotheses chart: across the top write, “How far can we skip rope in any given time?” Create two columns and label one “seconds” and the other “meters.”

If you have a student with low vision, consider giving them a handout of the chart with large print. The student's partner can assist with recording the data that is on the class chart onto the personal chart.

Part One: Introduction (10 minutes)

Note to Group Leaders:
When you introduce the activity, be aware that some students may be concerned about their rope-jumping skills. Assure everyone that this is not a competition. Explain that, for this activity, instead of jumping rope in place students will be "skipping" rope to maximize the distance between turns of the rope.

Call students together and ask, "Do you think we can predict how far (distance) a person can skip rope in a designated time?" "Can someone tell me what we mean by designated time?" Ask students to think about what "distance over time" might mean. Ask some questions to help students focus:

- Have you ever seen, or participated in, a sporting event where the athlete was timed? (You can prompt with examples like track or swimming.)
- How was distance measured?
- How was time measured?
- Students can generally connect to ideas like a four-minute mile. Chart their ideas of sports that record distance in relation to time.
- Ask each student to guess (make a hypothesis) about how much distance they can cover jumping rope in a designated time, e.g., three seconds. To count seconds, students can say "one Mississippi, two Mississippi," etc.
- Have students record their guesses on the "hypotheses" chart.

Meter measuring tapes can be made tactile for students who are visually impaired by use of string or colored masking tape placed at meter intervals. Using "flags" at the 10 centimeter points is also helpful for students with learning disabilities.

String with knots placed at various intervals (meter, decimeter, etc.) is also a useful piece of equipment to provide for students who are visually impaired.

Trundle wheels are wonderful tools for measuring distance and they provide both an auditory and tactile cue as they move along various distances. These tools are helpful for students with auditory, visual, and learning disabilities.


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Part Two: Skipping Rope in Pairs (20 minutes)

Students will work in pairs and choose a jump rope that is a good length for their height. If one person is taller than the other, students can select a rope that can be modified, e.g., by rolling the handles around their hands to make the rope shorter.

1. Divide students into pairs and let each pair select its own jump rope. Explain that each member of the pair will record how far their partner skipped rope and keep time.
    - If a student uses a wheelchair or has other mobility issues that preclude "skipping," assign three students to that team and allow the student's two partners to turn the rope so that the student can travel by walking or rolling a distance.

2. Give out paper and pencils and ask students to plan how they will record their data. For example, they can divide the paper in half and write each person's name on top and write "Distance" on one half and "Time" on the other.

3. Tell students that they will have 20 minutes to take turns skipping rope and then everyone will come back together, and each pair will plot their data on a line graph.

4. As the pairs skip rope, circulate to see if they have any questions. Give students a heads-up about five minutes before time is up, so that each person can have a final chance.

Part Three: Creating Line Graphs (10 minutes)

1. Distribute line-graph forms and explain that each pair of students will be responsible for creating a line graph to chart their information. Explain that the x axis (the horizontal axis) will indicate the time traveled (one second, two seconds, etc.). The y axis (the vertical axis) will indicate the distance (one meter, two meters, etc.). The students may want to label the x axis using multiples of a number, e.g., count by threes.

2. Hold up a completed line-graph model and review with students how each point is placed on the graph. Students may want to use a ruler to locate the points on their graph.

   - When each pair has completed their line graph, they can return to the general meeting area.
   - For students with visual impairments, chart paper with graph lines is available for a large sample of graphing. You can also make graphs tactile in several ways. Use regular glue and let it dry to create a raised line graph. Or use wax-covered yarn or pipe cleaners to create the graphs.

Photo by https://unsplash.com/@jcoudriet on Unsplash

Part Four: Sharing Findings (10 minutes)

1. Gather the students and ask if one pair would like to explain what their graph represents to the whole group. What does the line look like in the graph? Does it move upward? Does it move downward? Is it a straight line? What does it tell us about this experiment?

2. Give each pair of students a chance to describe their graph.

3. Refer back to the hypotheses chart. Ask students if the graph turned out the way they thought it would. Does the graph tell them anything else?

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4. Ask students to choose any time that falls within the range on the x axis. Can they find the value for that time? For example, if the Y value is two seconds, how far would the jumper have traveled in that time? Remind students that this is an estimate.

Part Five: Testing the Hypotheses/Predictions (10 minutes)

1. Ask students to review their line graphs and predict how far they would go if they jumped rope another time within the range on the x axis. Have students try jumping for that time and measure how far they went—were they able to predict the distance accurately?

2. Ask students to share their findings with the group. Did they get the results they expected? How were they different? The same? What are some possible explanations for their results?

3. Post students' line graphs around the room.

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Part Six: Literacy and Math Identity Activity (60 minutes)

1. Show students a plot diagram for a narrative arc (exposition, conflict, rising action, climax, falling action, and resolution). Note that plot diagrams look a little like a line graph.

2. Ask students to brainstorm ideas for a super jump rope story about a pair of jumpers that can cover ever greater distances in ever shorter times. Encourage students to use their imaginations to make this possible.

3. Connect some of the ideas they have to the points on the narrative arc plot diagram.

4. Give students time to draft their version of the story, using some of the ideas on the plot diagram.

5. Have students read each other's stories and give their feedback. Then give students time to revise their stories if they choose.

6. Ask students to create an illustrated version of their story and publish it using art materials, an online design program, or a desktop publishing program.

Additional Literacy and Math Identity Activity

If this activity better suits your students or the goals of your afterschool program, replace Part Six with it. Or add this activity into the math to provide extra math-identity building.

- Ask students to create a list of "time/distance" sports. One example might be horse racing where victory is always a matter of covering the designated distance in the shortest time. Sailboat racing, auto racing, track, and swimming meets are other examples. If students are not familiar with these sports, ask them to do research by asking family members or searching the Internet.

Equity

Students may have very different abilities and levels of experience when it comes to jumping rope. It is important that those students who have had less experience can learn from the more experienced students and that they have time to practice their skills. Students will become aware of the amount of practice that goes into becoming skilled at skipping rope and gain respect for students who display perseverance.

Reflecting on the Activity

- Were students respectful of each other's skipping-rope skills?
- Did students understand that this was not a contest?
- Were students able to interpret the information from their line graphs?

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

+4 Going to Great Lengths

+4 Going to Great Lengths

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Question

Is there a relationship between jump rope length and the height of the person who is jumping?

Objectives

Students will:

- Work in small groups to problem-solve.
- Use standard and nonstandard measurements.
- Observe the shape that the rope creates as it turns.
- Use multiples of numbers to represent data.
- Create a scatter graph to represent data.
- Observe patterns in their data.
- Use data to make predictions.

Where's the Math?

Students will use nonstandard and standard units of measurement to record the length of jump ropes and the height of individual jumpers. As they compare the different lengths of the rope to the height of the jumpers, they will look for a mathematical relationship between the two numbers called a ratio. They will use the data collected to create and interpret a graph and identify patterns in the data to help make predictions about the length of a jump rope needed for other individuals. Students will understand the appropriate use of a scatter graph as a representation of a trend in data.

Math Skills Developed

- Using nonstandard and standard units as measuring tools
- Problem-solving
- Recording data
- Graphing
- Understanding ratios and mathematical patterns
- Interpreting data to make predictions

Materials

- ☐ Clothesline or utility rope (~10mm diameter) cut into 10-foot lengths (one for each small group)
- ☐ Standard measurement tools (e.g., meter stick/ yardstick, etc.)
- ☐ Nonstandard measurement tools (e.g., feet, sneakers)
- ☐ Chart paper
- ☐ Color markers
- ☐ Color dots
- ☐ Pencils

Handouts:

- ☐ Data Chart—Nonstandard Units
- ☐ Data Chart—Standard Units
- ☐ Line-Graph Template B

NCTM Math Standards

Content Standards

- **Number and Operations:** Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- **Measurement:** Understand measurable attributes of objects and the units, systems, and processes of measurement.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through problem solving; apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematical problem solving.
- **Representation**: Create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena.

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Getting Ready

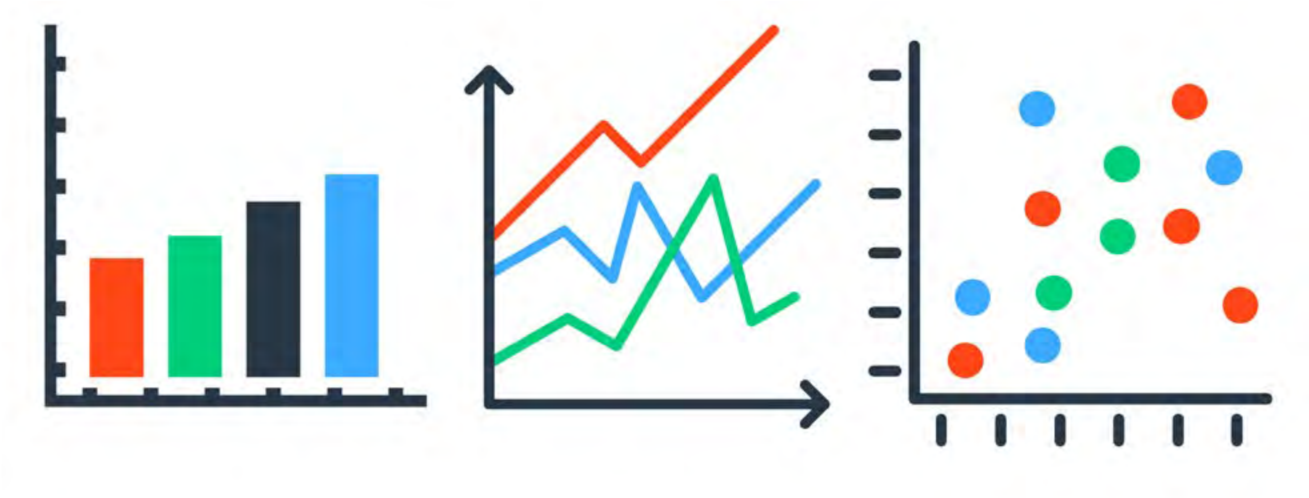
Be sure you have enough clothesline rope so that each small group will have a 10-foot length. If possible, arrange to conduct the activity in a large space like the gym, cafeteria, or outside.

Introduction

Students will try various lengths of jump rope until they find the length that is best for them. Working in small groups, they will use nonstandard and standard units of measurement to measure the rope as well as the height of the jumper. Then they will record their findings, use the data to create a graph and look for a pattern, and determine if the graph is useful in making a prediction.

It may be helpful for students to review the different types of graphs they have used in this theme before introducing scatter graphs. Review: A bar graph is used to show comparisons, and a line graph shows the relationship between two variables.

A scatter graph is used to identify trends when considering two variables. In this activity students will ask, "Is there a relationship between the height of the jumper and the length of the rope?" By plotting their data on a graph students will determine if there is a direct relationship between the variables or if there is a more general relationship or trend.



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Part One: Jumping Rope (10 minutes)

Divide students into groups of four to six. Give each group a 10-foot length of clothesline and have each student take a turn using it to jump. (Students may naturally begin to shorten the length of the rope.) After about 10 minutes of jumping, bring the group back together for a discussion.

Part Two: Large-Group Discussion (15 minutes)

1. Review the activities that you have been doing with students about jumping rope, e.g., the materials used in jump ropes and the reasons why people jump rope.
2. Ask students, "What do we mean by the 'best' length for a jump rope?" "Do you think it is the same for everyone?" "What criteria would you use to determine the best length?" Be sure that everyone understands that criteria are standards or rules by which a judgment (e.g., "best" length) can be made.
    - Ask them to explain their ideas. For example, "The best length would let me jump for two minutes without missing!" Or, "The best length would make it easier to jump."
    - Students may refer to the ropes that they just used and discuss how they adjusted them to get the best length. Encourage students to discuss their ideas fully. Would a jump rope need to be longer or shorter than the person jumping? You may want to have a student illustrate a rope that is too short and one that is too long. What happens when the rope is too short? Or too long?
    - If a student with a hearing impairment is present in the large-group discussion, have students who are speaking keep their hands raised for a few moments after they begin speaking so that the student can locate them. This helps with lip reading and filtering out extraneous sounds.

3. Ask the students if they think taller jumpers need longer ropes. Do shorter jumpers need shorter ropes? Do they think there is a relationship between jumper height and rope length? Ask the students to make predictions about this relationship. Try to categorize their responses (for example, no relationship between jumper and rope size, direct relationship, sometimes a relationship) and ask each student which they think will happen.

4. Chart their responses and label it "Hypotheses".

5. Explain to students that they will be working in small groups to measure the length of a jump rope and the height of the jumper. They need to decide as a whole group which unit of measure they are going to use (e.g., centimeters, meters, inches, or feet). Point out that these are standard measurements. Ask, "If we didn't have access to rulers or measuring tapes, what else could we use to measure?" Make a list of possibilities (e.g., sneaker lengths, hand widths). Explain that these are nonstandard measurements. Have each group select a nonstandard measure to use as well.

Note to Group Leaders:
For the data to be uniform, it is important that every group use the same standard measure they decided on during the whole–group discussion. However, each group can have fun with a different nonstandard measure. Encourage creativity! While it may seem obvious to an adult, it may not be so obvious to students that the same unit of measurement (whether standard or nonstandard) should be used for the student and the rope length. If students are measured in ice pop craft sticks, then the rope should be measured in ice pop craft sticks. If students are measured in centimeters, then the rope should be measured in centimeters.

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Part Three: Small-Group Investigations (20 minutes)

Note to Group Leaders:
To collect enough data for the graph to show a trend, every student should be measured. Each student should have a turn measuring and recording the data on the data chart. Encourage students to take turns recording and reporting.

1. Have students assemble back in their small groups and give each group two data charts (one labeled Standard Units of Measure; one labeled Nonstandard Units of Measure), along with a 10-foot length of clothesline rope.

2. Ask students to measure each jumper's height first using the standard measure that was agreed upon and record it on the Standard Units data chart. Ask students to measure each other from their foot to their knee, knee to hip, hip to shoulder, and shoulder to the top of their head. Ask them to add these four measurements together to calculate their total height.

3. Next, have students measure using the nonstandard measurement tool and record it on the Nonstandard Unit data chart. Remind students to record the name and height of the person being measured on the charts, along with the units of measure.

Note to Group Leaders:
Height can be a very touchy subject for many students and is of particular concern for students who use wheelchairs, who often are viewed as short because they are seated. By measuring the students in segments (e.g., knee to hip) every child's full height can be measured, and they will have additional practice with measuring and adding.

4. Ask each jumper to jump rope with the clothesline, adjusting the length until they think it best for them.
   - Again, students with mobility issues may feel excluded here. Be sure to stress that they are simply giving their guess as to what is or would be best for them. Let them take the lead as to how that will be determined.
   - Another possibility would be for two students to hold the rope on either side of (and in front of) the student as if they were going to roll over the rope. Then measure the length of that rope.
5. Have students measure the length of the rope, with the standard and nonstandard unit and record it on the data chart next to the student's height.
6. Have students rotate roles, until everyone has a turn being measured and jumping, using the same standard and nonstandard unit of measurement for each person.

Part Four: Sharing Findings and Literacy and Math Identity Activity (20 minutes)

Bring the students back into the large group. Ask each small group to report on their findings, using the charts to illustrate the data they collected.

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1. Begin by discussing the standard and nonstandard measures they used. Do they see a relationship between the two? Which was easier to use? Why? Why do students think there are standard measures? Point out that nonstandard measures can change over time (for example, a person's foot will get bigger; hands are different sizes).
2. Read *How Big Is a Foot?* written and illustrated by Rolf Myller (New York: Random House Children's Books, 1991), a humorous look at the need for standard measurements. Ask students how the story connects to their thoughts about why standard measures exist.
3. Then ask students how they think they could organize their information in a way that helps them to see a pattern. For example, one way is to organize the measurements is from least to greatest, or from greatest to least. Using another data chart, record the name of the shortest student, along with their height and rope length. Continue in size order until everyone's data is recorded. Ask students what patterns they see. By organizing the data did they create a pattern?

Part Five: Creating a Scatter Graph (15 minutes)

Explain to students that, using the data from their Standard Unit charts, they are going to create a graph, which is another way to record the information they gathered.

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1. Using the template, help students label "Student Height" as the x axis and "Rope Length" as the y axis. A graph is created by finding a value for either axis and then finding the corresponding value on the other axis.

   - If the jumper's height is 150 centimeters (5 feet), find 150 centimeters on the x axis (Student Height), then find the value for rope length (e.g., 240 centimeters, or 8 feet) on the y axis. Follow the lines for those values until they intersect. Place a colored dot on that spot.

2. Repeat for each student.

Note to Group Leaders:
It is sometimes helpful to use multiples of numbers to indicate a unit on a graph. For example, if height was measured in inches and each unit on the graph represents an inch, the graph would have to be enormous! Instead, a multiple can be used. If each unit represents 10 inches, then the graph can be 10 times smaller. It may be helpful to have prepared examples of the graphs using different multiples as units on graphs. For example, one graph might use 5 inches per unit, and another might use 10. This provides a visual aid for students to see the difference in graph size as the units change.

3. When all the dots are in place, ask students if they see a relationship in the data. Do most of the dots representing tall jumpers also represent longer jump ropes? Do most of the dots representing shorter jumpers also represent shorter rope lengths? Students will likely see that most of the dots don't fall on a straight line but are scattered around an imaginary line from the first few dots to the last few. This is called a scatter graph.


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4. Ask students to draw a line (in pencil) from the tallest jumper to the shortest jumper. Do most of the other dots cluster around this line? Can the students draw a line that most of the dots cluster around? Although all the dots don't fall on the line, they show a trend or generalization.

   - For example, generally, a taller person will need a longer jump rope. Tell students that the dots farthest from the line are called "outliers" and represent jumpers who don't fit into the pattern. Ask students to discuss why the outliers are unique. Are some of the jumpers especially good athletes? New jumpers? Jumping the rope in a different way? Ask students to think about other examples of outliers. (For example, if the students collected data on how fast people could run, Olympic runners would be outliers.)

5. Ask students if their data shows a relationship between student height and rope length. Refer to the chart with their hypotheses. What does their graph tell them? Can students use the information in the scatter graph to predict what length a rope jumper who is 6 feet tall would need? What information does the graph convey?

Note to Group Leaders:
It is important for students to realize that gathering data is not about looking for a "right answer." If the data they collect does not match their hypothesis (e.g., the taller the jumper, the longer the rope), that is OK. They should not try to make the data "fit"--instead, they need to analyze the data, consider why the outcome may have differed from their expectations, and perhaps revise their hypothesis. This is what scientists and mathematicians do all the time!

6. Remind students to write down any questions they have on the "Our Questions about Jumping Rope" chart.

Additional Literacy and Math Identity Activity

Add this activities into the math to provide extra math-identity building.

- Have students create a group story about a "giant who wants to jump rope." Ask questions to get them started, such as "How tall is the giant?" "What would a giant use as a jump rope?" "How would you describe the sound of a giant jumping rope?" Then have students write a comparison story about an "elf who wants to jump rope." What if the giant and elf were friends and wanted to jump rope together?

Equity

It is important to think about what would be most equitable and fair in assigning roles to students for small-group work. Be sure that all students are able to actively participate. Be sure to address any teasing that might occur over being short or being tall; make clear that this is not a competition about "who's the shortest" or "who's the tallest."

Reflecting on the Activity

- Did each student have a turn to jump and be measured?
- Did students understand the difference between standard and nonstandard measures?
- Were students familiar with line graphs? Scatter graphs?
- Is there additional information students need to understand a line or scatter graph?
- Did students understand that they were collecting data, not looking for a right answer?

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

+5 Designing a Mathematics Investigation

+5 Designing a Mathematics Investigation

Question
How can we design an investigation to answer our questions about jumping rope?

Objectives
Students will:

- Pose a question related to Jump Rope Math.
- Design an investigation.
- Collect data pertinent to the investigation.
- Represent data using a graphing (bar, line, scatter, Venn diagram) format.
- Interpret the data to answer their question.
- Share their findings with other groups.

Where's the Math?
As the students collect data for their question they will chart, graph, and represent their data in different ways. Students will then interpret the data to answer their real-life question. (In this activity mathematics truly becomes the language of science!)

NCTM Math Standards

Content Standards

- **Data Analysis and Probability:** Formulate questions that can be addressed with data; collect, organize, and display relevant data to answer them.

Process Standards

- **Communication:** Organize and consolidate mathematical thinking through communication.
- **Problem Solving:** Build new mathematical knowledge through problem solving.
- **Reasoning and Proof:** Make and investigate mathematical conjectures; develop and evaluate mathematical arguments and proofs.
- **Representation:** Create and use representations to organize, record, and communicate mathematical ideas.

Math Skills Developed
- Designing an investigation
- Collecting data
- Representing data
- Constructing graphs
- Interpreting graphs
- Problem-solving
- Reporting data

Materials
The materials required to conduct the investigation will depend on the question that students want to answer. However, the following general materials will likely be needed:

- ☐ Sticky notes
- ☐ Graph paper
- ☐ Chart paper
- ☐ Markers
- ☐ "Our Questions about Jumping Rope" chart

Handout:
- ☐ Investigation Planning Form

Getting Ready
During this theme students have been writing down their questions about jumping rope. Now, they will have the opportunity to find their own answers! Give students a few minutes to review the questions that were recorded during the theme. What do they want to know? Do other questions come to mind as they review the chart? To help students organize their investigation, make a copy of the Investigation Planning Form hand-out for each group.

This activity is different from the other activities in this theme because it requires the students to plan their own investigations. In the first four activities, students developed the skills and abilities to collect, record, and process data. Now they are challenged to design their own investigation, putting all these skills to use.

First, they must think about the question they would like to ask. For example: Does the surface of the floor make a difference in how long a student can jump? Does a person jump rope better in sneakers or barefoot? Does singing a song or repeating a rhyme increase the number of jumps a person can do? Can a person jump longer if other students are turning the rope?

Some questions will be more inclusive than others for students with disabilities. For example, does having your eyes closed affect the number of jumps a person can do might be a great question for a group with a student who is blind or has low vision; it would encourage teammates to experience jumping rope the way this student does. On the other hand, determining whether having a rhyme or song sung impacts the number of jumps may not be a good choice for students who are deaf or hard of hearing.

Part One: Large-Group Discussion (15 minutes)

1. Review the questions students have written on the "Our Questions about Jumping Rope" chart. Ask students if there are any other questions about jumping rope that interest them. Add them to the chart.

2. Explain to students that they will work in small teams to answer the questions by a) designing an investigation process, b) collecting data, c) charting and graphing their data, and d) interpreting the data.

3. Review the various kinds of graphing students accomplished during the first four activities: bar graphs, line graphs, scatter graphs, Venn diagrams.

4. Form small groups of four to six. Review the questions and ask each group to volunteer to investigate one question.

5. Distribute and briefly review the Investigation Planning Form with students:

   - Names of people in our group
   - What do we want to find out?
   - What is our research process?
   - What equipment do we need?
   - What role will each person have?
   - How will data be collected?
   - How will data be recorded? (Bar Graph? Line Graph? Scatter Graph? Venn Diagram? Other?)
   - How will we report to the whole group?

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Part Two: Planning the Investigation (15 minutes)

1. Ask each group to plan an investigation that will help them find the answer to their question.

2. Circulate during the planning process. Ask students for ideas about how they can accomplish their task. For example, if the students are interested in finding out if the surface of the floor affects the jumper:

    - What kinds of surfaces are available for jumping? Gym floor? Sidewalk? Carpet?
    - How many students will jump and contribute data?
    - How many times will each student try the different surfaces?
    - Who will record the data?
    - How will the data be recorded? Will students use bar graphs? Line graphs? Another method?
    - What else do students think needs to be included?

3. As students plan, ask them to make a prediction about the outcome of the investigation.

Part Three: The Investigation (45 minutes)

Using their Investigation Planning Form as a guide, have students answer their chosen question. The investigation will most likely be conducted over several days and involve the larger community—classmates, families, other students in the afterschool program.

For example, at the Devers Community Center at Fort Knox, a group of students conducted an investigation looking at whether the type of shoe worn by the jumper mattered. Their hypothesis was that jumpers wearing sneakers would be the best jumpers since they felt that sneakers offered the best cushioning, support, and traction. They surveyed their friends and families—many of whom are in the army. They found that people who wear combat boots (part of some of their parents' uniforms) were the best jumpers.

This finding led to an excellent discussion of the validity of their result. Do combat boots offer more support, cushioning, and traction? Possibly—but they hypothesized that perhaps the real reason for the unexpected outcome is that people in the Army are more physically fit and therefore better jumpers! The students decided that in order to find out they would have to conduct their investigation again—but with their parents wearing other types of footwear.

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Part Four: Sharing Findings (20 minutes)

1. Ask each small group to report its findings:

    - Were the expected outcomes achieved?
    - Was the outcome different than the prediction?
    - What are some ideas that describe the results you received?
    - Did this investigation lead to other questions?
    - Did more than one group investigate the same question? Were the results the same or different? Why might this have happened?
    - Was the data presented in the same or different ways?
    - Could this have affected the interpretation of the data?

Part Five: Literacy and Math Identity Activity (20 minutes)

Ask the students to plan a fantasy investigation – this will include a problem that has occurred and possible ways to solve the problem. It can be on another planet or in another universe. Let their imaginations take flight. How would this investigation work if it was in outer space? Or with aliens?

Additional Literacy and Math Identity Activity

If this activity better suits your students or the goals of your afterschool program, replace Part Five with it. Or add this activity into the math to provide extra math-identity building.

- Ask the students to think about an investigation that would answer a question that they have about something in their lives.

Equity

- Did all the students feel comfortable offering ideas for possible investigation questions?
- Did they feel their ideas were welcomed?
- Did the plan for the investigation include a role for every student?

Reflecting on the Activity

- Did students think that their questions had been successfully answered?
- Was every student encouraged to participate?
- Were the students familiar and comfortable with the data collection and interpretation process?
- Did the students feel good about their accomplishment?

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

+6 Career and Role Model Connections

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Career and Role Model Connections

Tahani Amer, Monica Barnes, and Melanie Wood are three young women from diverse backgrounds who think math is exciting and use it in their everyday lives and work. The short bios and activities that follow that can inspire students to think about math in new ways.

1. **Tahani Amer** is an aerospace engineer at NASA. She works in the Aerodynamic Measurement Branch and has many interesting tasks as a mathematician. She develops instruments and sensors that measure the parameters needed to design new airplanes.

2. **Monica Barnes** is an electronics engineer and aerospace technologist at the Langley Research Center in Hampton, Virginia. She works in the Systems Development Branch, a part of the Airborne Systems Competency. Her job is collecting data for research and industry partners at NASA. She travels to many exciting destinations to gather data.

3. **Melanie Wood** was the first woman to win the U.S. Mathematics Olympiad and to be on the U.S. Mathematical Olympiad Team for the International Math Olympiad. She has traveled to Asia and Europe for competitions.

Make copies of the biographical information on Tahani Amer, Monica Barnes, and Melanie Wood for students. Ask for a volunteer to read the biographies aloud.

- Tahani Amer, Monica Barnes, and Melanie Barnes each had to solve a problem. With students, make a list of some careers that involve problem solving. Identify the problem and discuss what skills would be needed to find a solution. Students may need help making this list and finding out about the careers. Together think about ways to find out this information. For example, information may be available on the Internet, at the school library, or from professionals already in this field.
- Make a list of math careers like Melanie Wood's that might include travel or meeting new people.
- Help students find the NASA website and search for information about careers. Ask students to make a list of careers that interest them.
- Have older or stronger readers choose one of the women in Wonder Women of Science by Tiera Fletcher, Ginger Rue, and Sally Wern Comport (Somerville, MA: Candlewick Press, 2021) to read about (or have students work in small groups). Have them read the brief biographies, search online for additional information, and put together a presentation for the rest of the group about their scientist.
- Have students research Patricia Walsh, an engineer who is both blind and a triathlete, online to find out she balances her technological career and her athletic pursuits with low vision.
    - Sample article: "The Amazing Story of the Blind Engineer (Who Is Also a Triathlete)" by Gwen Moran on Fast Company (December 4, 2014).

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

+7 Handouts

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HANDOUT

Jump Rope Math Role Model Biographies

Tahani R. Amer
"If I did it, anybody can. Believe in yourself!"

Tahani R. Amer is an aerospace engineer at NASA. She works in the Aerodynamic Measurement Branch developing instruments and sensors that measure the parameters needed to design new airplanes.

Math was Amer's favorite subject in school, and it still is. She enjoys trying to find new methods to solve work problems by using math models.

When Amer came to the U.S. from Egypt in 1983 and took her first calculus class, she received an A grade even though she could not speak a word of English. Her success helped Amer expand what she thought she might do for a career. She liked the idea of engineering and thought it was an exciting way to use mathematics.

The advice Tahani Amer has for young people is to look at the "picture" of her life. She had a different culture and language that made it harder for her to go to college and succeed in the U.S. She now has a master's degree in engineering and an exciting career at NASA.

Read more about Tahani Amer in her profile at NASA.

Monica Barnes
"Remember if you can believe it, then you can achieve it!"

Monica Barnes is an electronics engineer and aerospace technologist at the Langley Research Center in Hampton, Virginia. She works in the Systems Development Branch, a part of the Airborne Systems Competency. Her job is to collect data for research and industry partners at NASA. She likes the process of identifying and solving problems, applying them to real-life situations, and working as a team member to accomplish a goal.

In high school, math and science were Barnes' favorite subjects. She knew she wanted to explore a technical field but wasn't quite sure what. Her father suggested that she go into engineering. He also told her that it would be a challenge, requiring lots of hard work and dedication. Her mother guided her to explore this career field by encouraging her to attend the Minority Introduction to Engineering Program at Virginia Tech during her junior year.

Barnes is a creative person and also enjoys creative writing. She combined her interests and wrote a technical paper in college on one of NASA's co-op projects, which won a first-place prize at a National Technical Association's (NTA) undergraduate conference competition in 1983. This paper was later published in the NTA Journal.

In addition to her extremely busy work schedule at NASA, Barnes also participates in mentoring young people, especially young women, through the Cooperating Hampton Roads Organizations for Minorities in Engineering (CHROME) program.

Watch Barnes discuss her career.

Melanie Wood
"Insight, originality, inspiration, new perspectives, opening your mind, finding a different way, playing around. That is mathematics!"

Melanie Wood is the first woman to win the U.S. Mathematics Olympiad and to be on the U.S. Mathematical Olympiad Team for the International Math Olympiad. She was only 14 at her first competition.

Wood's love of mathematics started in childhood. Her mother would give her math problems to solve as a game! In the summer of 1996, Wood was thrilled to find a group of students at the Olympic Training Camp who saw mathematics the way she did: "I had finally found peers. Before, I'd always thought maybe I was just weird."

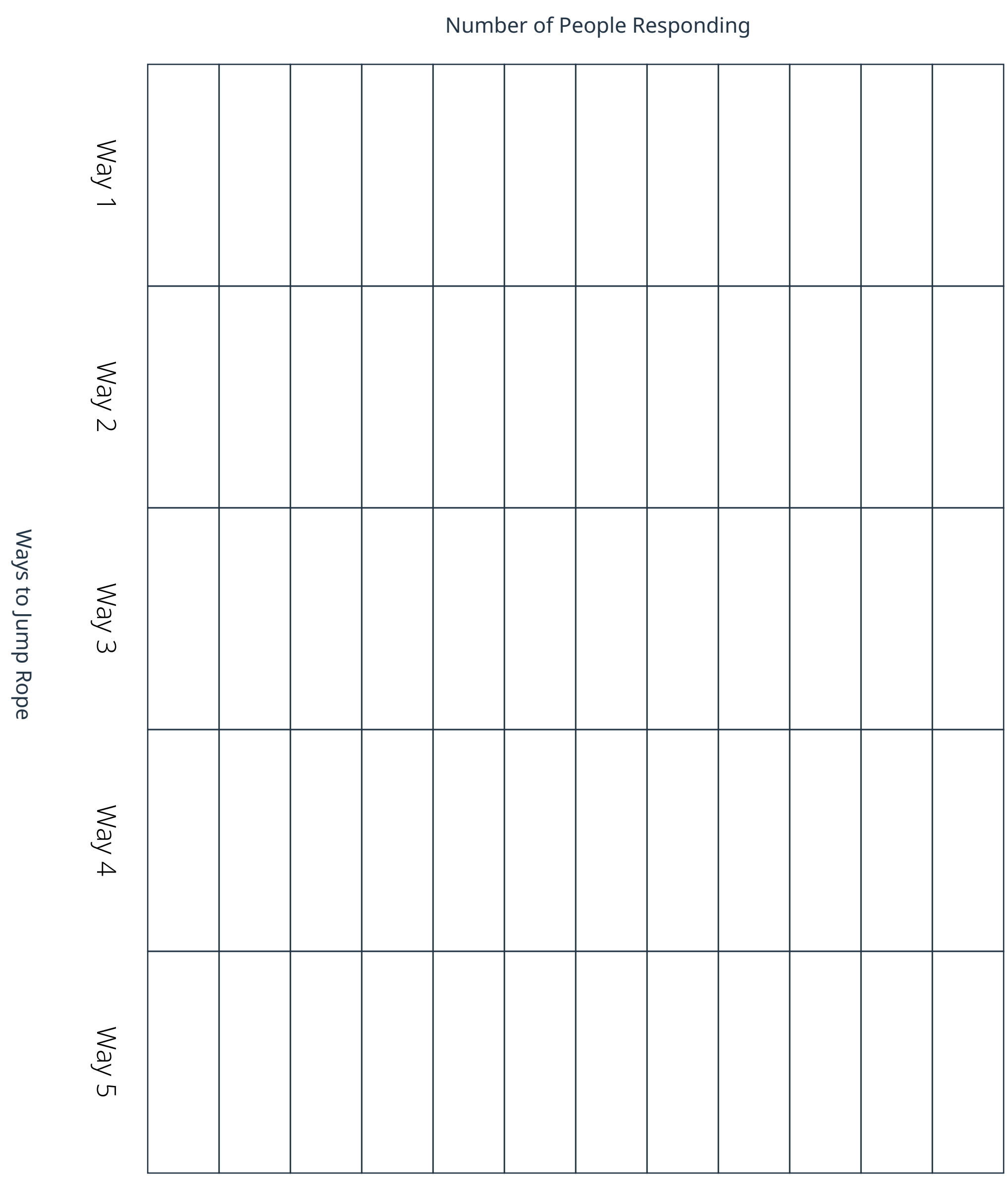
In high school, after Wood won the U.S.A. Math Olympiad, she realized that winning shocked her. When she wondered why she was surprised, she realized that her image of who won the competition was of boys. At Duke University, one of her teachers was the only woman tenured faculty in the math department. She was happy and having a woman in the math department was important to her. Because she understands what it is like to be a woman in professional mathematics, Wood intends to help other women. "I think that it's easier for a girl to see me and say, 'Oh, I want to be like she is.'

Wood is the first woman to win the Morgan Prize for Outstanding Research in Mathematics and the first U.S. woman to be named a Putnam Fellow—one of the top five scorers in a prestigious national mathematics competition.

Activity +1

Small-Group Investigation Bar Graph Template

How many ways do we know to jump rope?



- ☐ Way 1
- ☐ Way 2
- ☐ Way 3
- ☐ Way 4
- ☐ Way 5

ASM+
HANDOUT

Large-Group Investigation Bar Graph Template

What materials are used to make jump ropes?

Number of Jump Ropes

Materials					
Material 1					
Material 2					
Material 3					
Material 4					
Material 5					

- ☐ Material 1
- ☐ Material 2
- ☐ Material 3
- ☐ Material 4
- ☐ Material 5

Sample Survey: The Whys of Jumping Rope

1. Do you jump rope?

☐ Yes
☐ No

2. Why do you jump rope?

☐ For exercise
☐ Because it's fun
☐ Other (please list all the other reasons):

Sample Survey: The Whys of Jumping Rope

1. Do you jump rope?

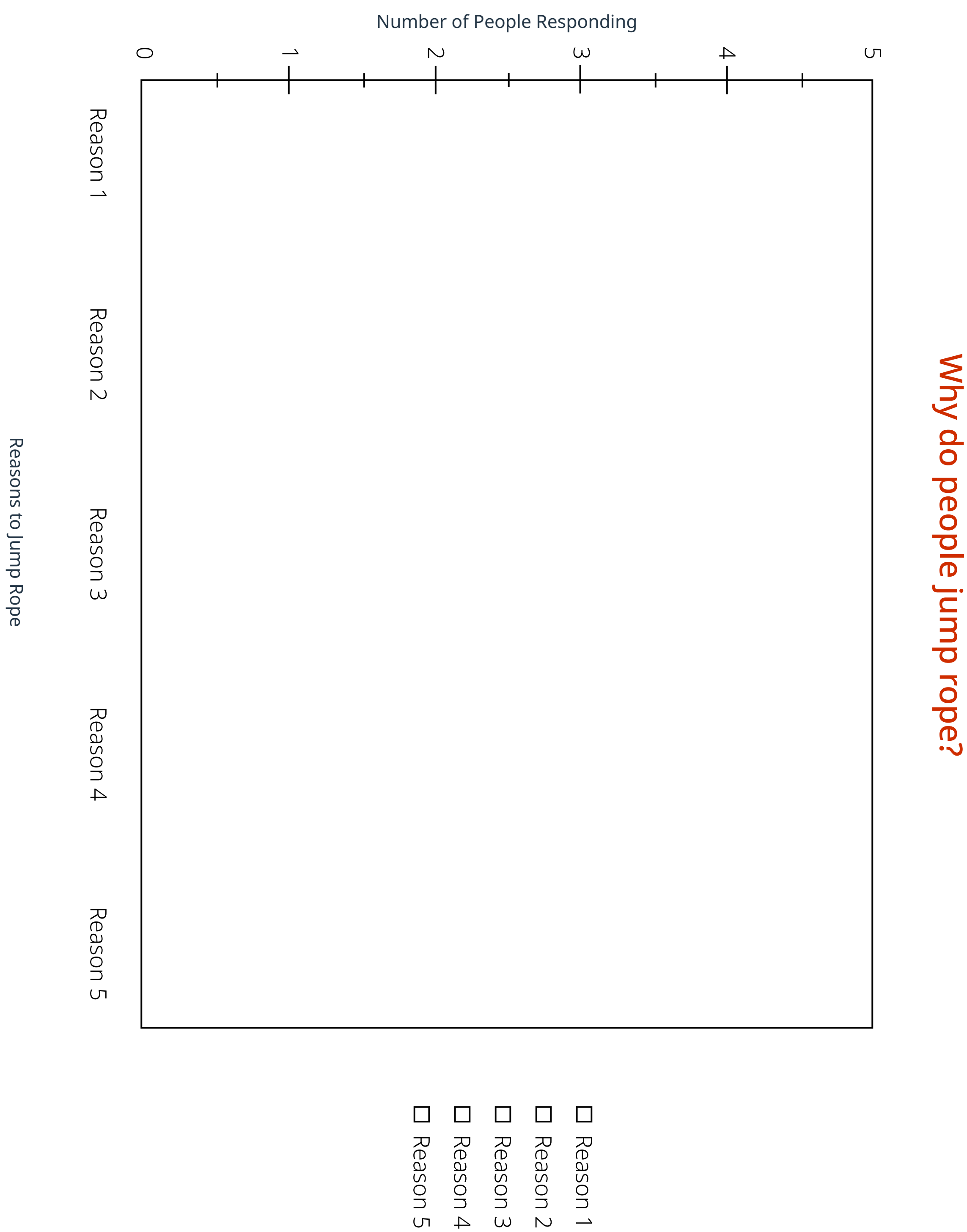
☐ Yes
☐ No

2. Why do you jump rope?

☐ For exercise
☐ Because it's fun
☐ Other (please list all the other reasons):

ASM+
HANDOUT

Bar Graph Template



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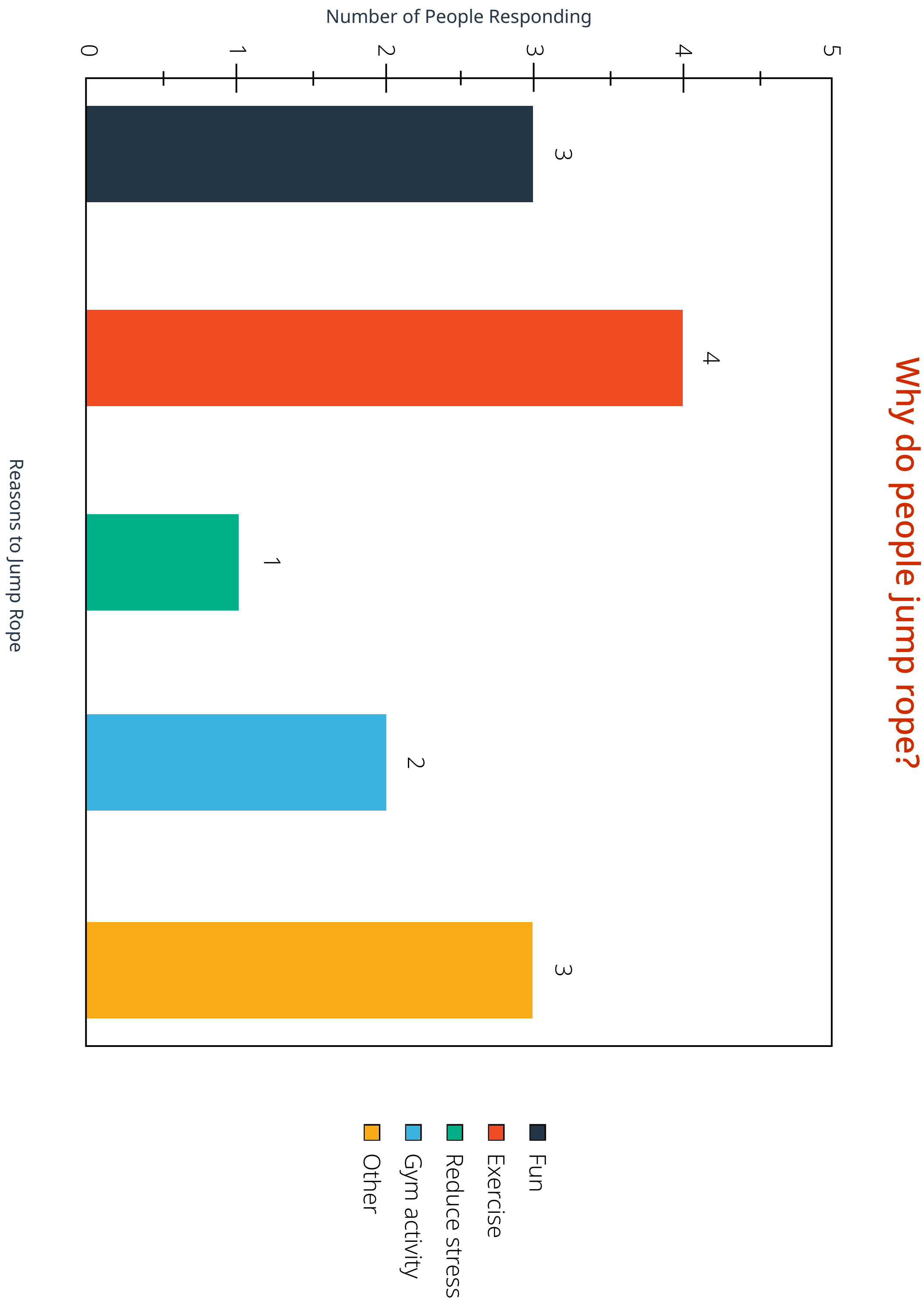
HANDOUT

Sample Bar Graph



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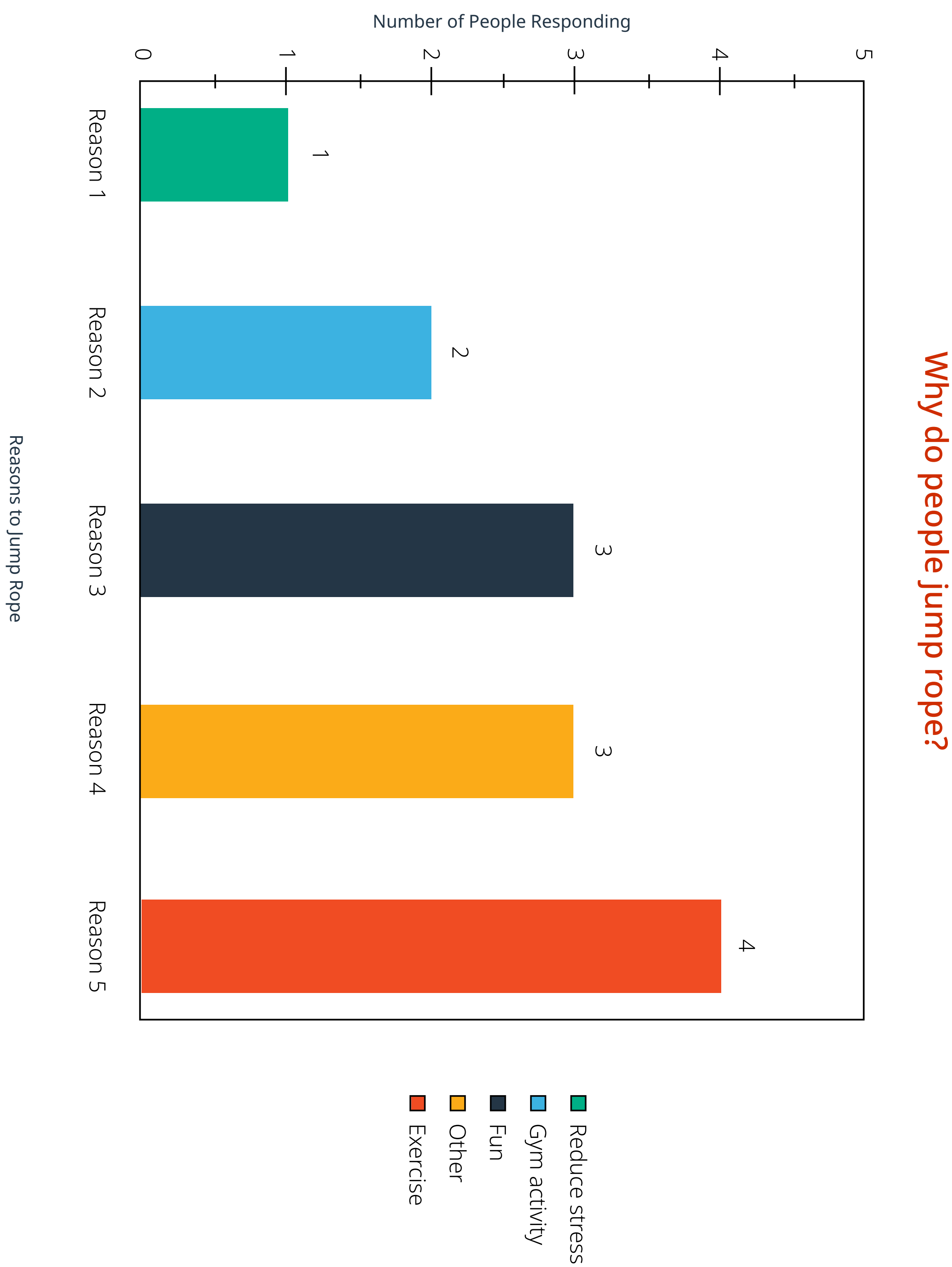
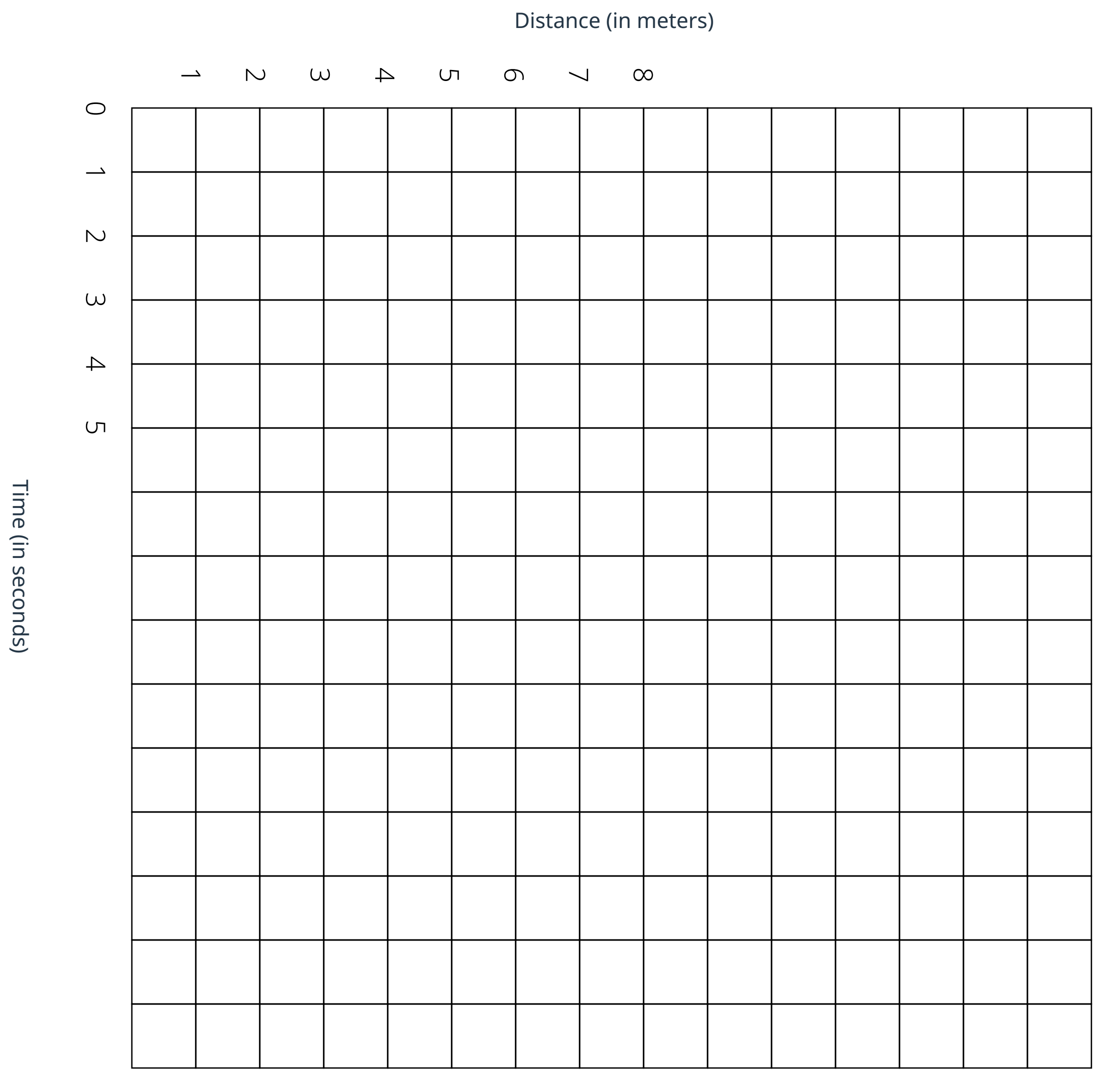
HANDOUT

Sample Bar Graph 2



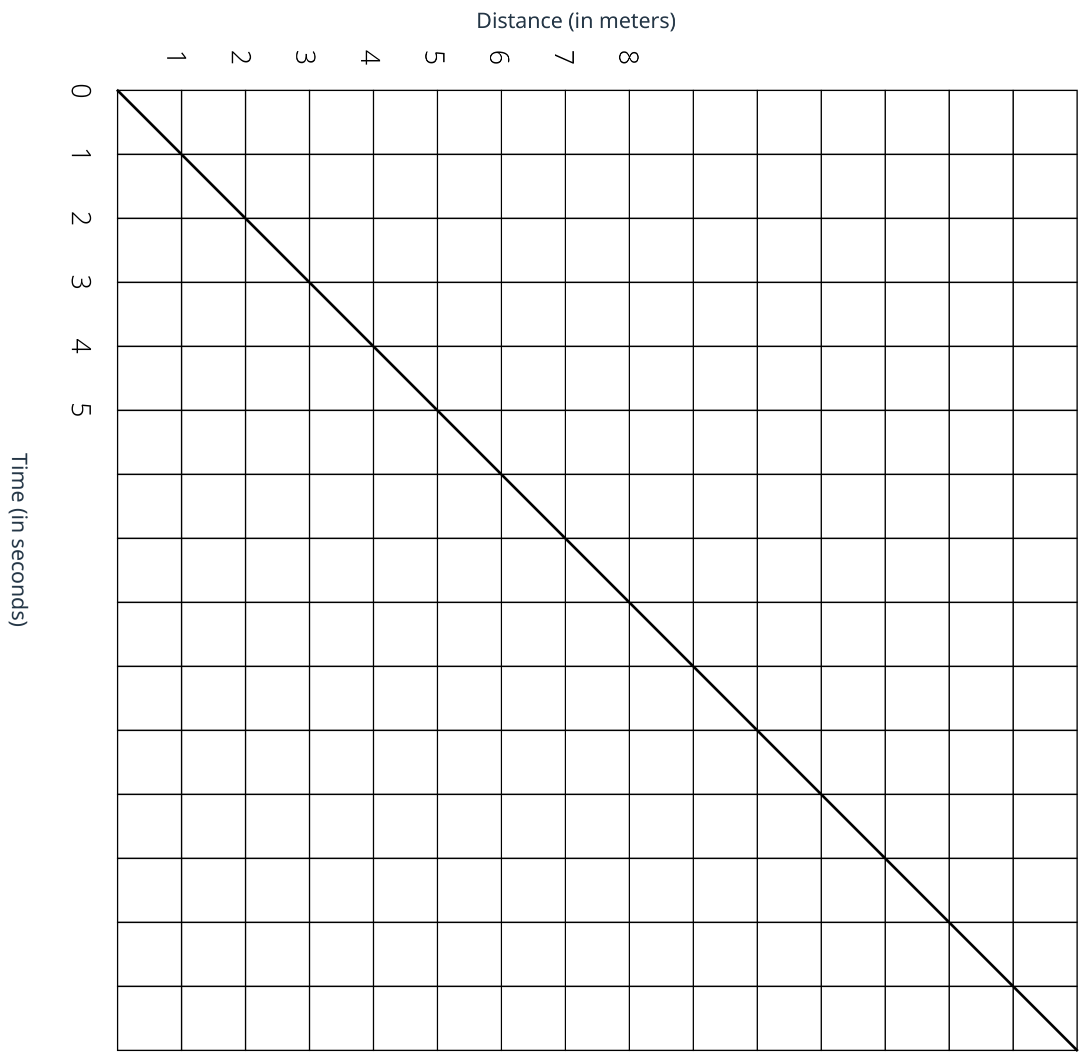
ASM+
HANDOUT

Line Graph Template A



ASM+
HANDOUT

Sample Line Graph



Nonstandard Units Data Chart

*Please indicate the units used for measurement, e.g., floor tiles, sneaker lengths, ice pop stick lengths, or hand widths.

Student Name	*Student Height (Nonstandard Units)**	*Rope Length (Nonstandard Units)**

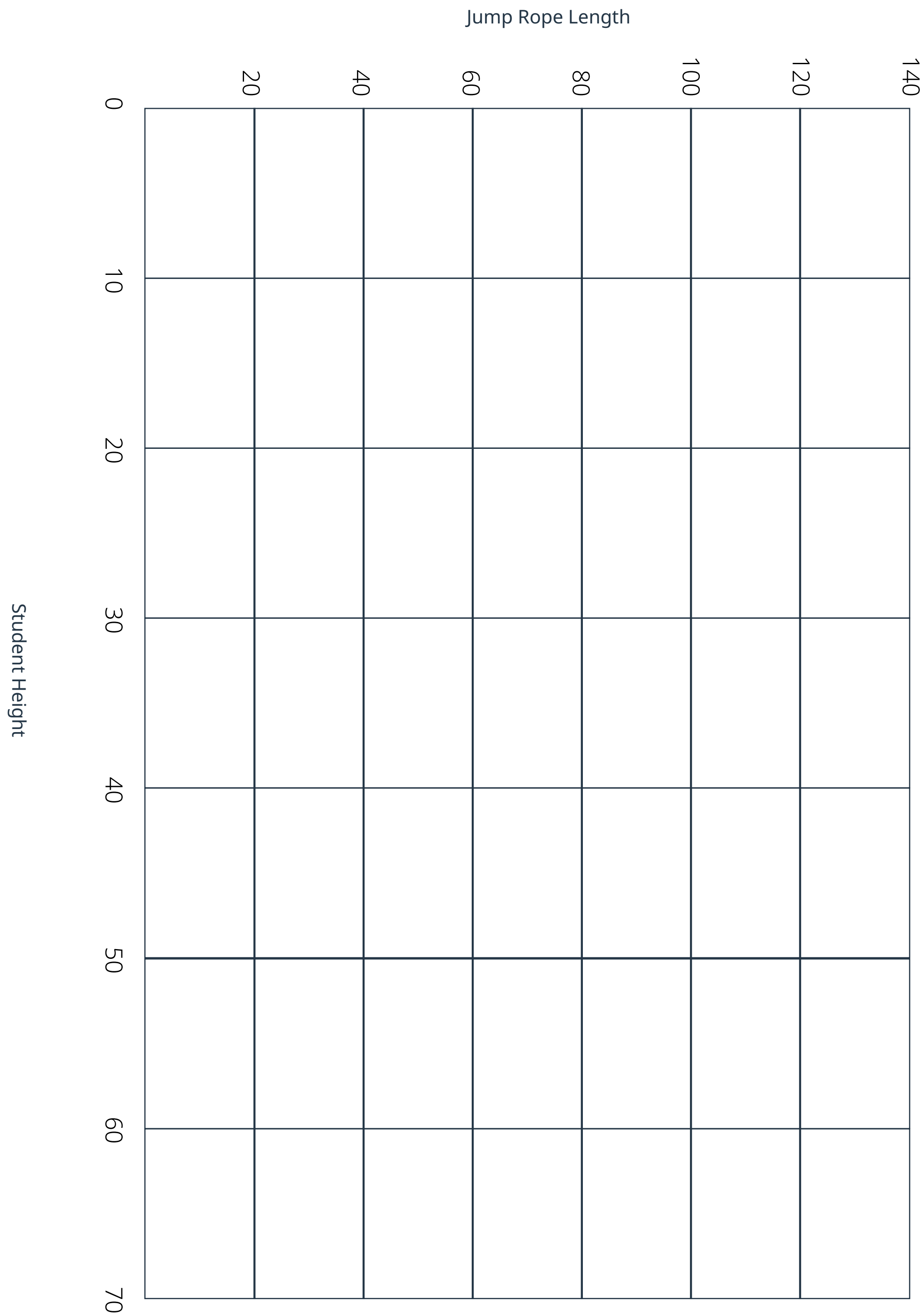
Standard Units Data Chart

*Please indicate the units used for measurement, e.g., inches, feet, centimeters, or meters.

Student Name	*Student Height (Standard Units)**	*Rope Length (Standard Units)**

ASM+
HANDOUT

Line Graph Template B



What is the relationship between jumper height and jump rope length?

Investigation Planning Form

1. Names of people in our group:

2. What do we want to find out?

3. What is our research process?

4. What equipment do we need?

5. What role will each person have?

6. How will data be collected?

7. How will data be recorded?

8. How will we report to the whole group?

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

+8 Resources

Books

Double Dutch by Sharon Draper (New York: Simon and Schuster, 2004). The story of three eighth-grade friends, preparing for the International Double Dutch Championship jump rope competition in their home town of Cincinnati, Ohio, who have to cope with Randy's missing father, Delia's inability to read, and YoYo's encounter with the class bullies.

How Big is a Foot? by Rolf Myller (New York: Random House Children's Books, 1991). The King wants to give the Queen something special for her birthday, but the Queen has everything except a bed. Because beds had not yet been invented, no one in the Kingdom knows the answer to a very important question: How big is a bed? This humorous book poses a question that provides the reader with a chance to problem solve using nonstandard units.

Solo Girl by Andrea Davis Pinkney (New York: Hyperion Books for Children, 1997). The story of Cass, a girl who would like to fit in with the girls in the neighborhood who do fancy rope jumping in their spare time. Although she is good at math, Cass can't seem to get her feet to work as well. The story tells about Cass's attempts to become an accepted part of the neighborhood, and her courage in mastering a difficult task.

Websites

Association for Women in Mathematics
AWM is a nonprofit organization dedicated to encouraging and promoting women and girls in the mathematical sciences including pure mathematics, statistics, and applied math.

Kids' Zone Create a Graph
This website provides the tools to create different kinds of graphs (line, bar, area and pie) that can be printed when completed. A great resource for older students.

Double Dutch Divas
Meet the Double Dutch Divas! They are a group of women in their 30s, 40s and 50s who use jump ropes, music, and imagination to have fun. They have performed their jump rope "dance" in Italy, Canada, and the United States at fairs, festivals, and special events. They create all their own routines and write original songs for their performances.

International Jump Rope Union
The IJRU is a global rope skipping organization. This site has rules, competition, and pictures of rope skippers from around the world.

NASA
Visit NASA's website to explore careers. The site also has information on Women in STEM careers.

WomenDoMath
WomenDoMath is an easy-access platform supported by the NSF that brings you news and resources about women and other groups or individuals in mathematics that accurately represent the diversity of the math community. WDM aims to provide a central hub and resource pool within reach for women in mathematics. It can also serve as a means of learning and growth for anyone wanting to promote gender equity and overall inclusiveness in STEM. Use this portal to connect with others, check out upcoming math events, and stay up to date with this and more communities in mathematics.

Afterschool Math Plus

Revised Edition

Theme +3: Jump Rope Math

+9 Glossary and Materials List

GLOSSARY

Axis A real or imaginary straight line used as a reference in a graph.

Bar Graph A way of displaying data using horizontal or vertical bars so that the height or length of the bars indicates its value.

Centimeter 1⁄100 of a meter.

Chart A sheet giving information in a tabular form.

Coordinates An ordered pair of numbers or letters used to show position on a grid. Coordinates are a way of describing a location in relation to the X and Y axes in a line graph.

Data Factual information (as measurement or statistics) used as a basis for reasoning, discussion or calculation.

Graph A visual way of representing information that makes it easier to understand.

Hypothesis A tentative assumption made in order to draw out and test its logical or empirical consequences.

Line graph A graph in which points (coordinates) are connected by a line or line segments to represent data.

Mathematical relationship A way of describing the relationship of two or more things using numbers, words or symbols.

Mathematics A study of relationships among numbers, shapes, and patterns. Mathematics is used to count and measure things, to discover similarities and differences, to solve problems, and to learn about and organize the world.

Measurement The act or process of measuring, or finding out "how much." It is a geometric function that describes the distances between pairs of points in a space.

Meter International standard unit of length, approximately equivalent to 39.37 inches. It was redefined in 1983 as the distance traveled by light in a vacuum in 1/299,792,458 of a second.

Nonstandard units of measurement A way of quantifying measurement using objects from the environment, such as a stick, one's hands, floor tiles, etc.

Parabola A curve formed by the intersection of a circular cone and a plane parallel to an element of the curve.

Pattern A set of shapes or numbers that are repeated in a predictable way; patterns may often be described by a rule, using words or variables.

Perpendicular Intersecting at or forming right angles (90 degree angle). Prediction – A reasonable guess.

Ratio Comparison of two quantities with like units. Set – Any group or number of things.

Scatter graph A scatter graph (sometimes called a scatter plot) uses paired data to plot points on a coordinate plane (a graph with x and y axis). A scatter plot is used to show a relationship between two variables (like jumper height and rope length) that is not a direct (linear) relationship.

Standard units of measurement A way of quantifying measurement using the English, metric, or other recognized system of measurement.

Statistics The branch of mathematics that deals with the collection, organization, and interpretation of data. A baseball player's batting average is a statistic.

Survey The gathering of information or data. The result of a survey is statistics. When you ask how many students have brought jump ropes for the math activity, you have taken a survey.

Venn Diagram A way of illustrating relationships between two or more sets that uses intersecting circles.

Materials List

- ☐ Bags (recycled plastic or paper)
- ☐ Chart paper
- ☐ Clothesline or utility rope (~10mm diameter) cut into 10-foot lengths (one for each small group)
- ☐ Color dots
- ☐ Extra ropes made from a variety of materials
- ☐ Graph paper
- ☐ Jump ropes
- ☐ Markers
- ☐ Meter measuring tape
- ☐ Nonstandard measurement tools (e.g., feet, sneakers)
- ☐ "Our Questions about Jumping Rope" chart
- ☐ Paper
- ☐ Pencils
- ☐ Rulers
- ☐ Standard measurement tools (e.g., meter stick/yardstick, etc.)
- ☐ Sticky notes

Afterschool Math Plus

Theme +4: MusicMath

Canva

Introduction

Students enjoy music and listen to it every day, but they are probably unaware of how much math and music are related. In ancient Greece and up to medieval times, music was considered part of mathematics! Musicians concentrated their efforts on understanding the mathematical basis of tones. One 18th century philosopher wrote: "Music is a secret arithmetical exercise, and the person who indulges in it does not realize that he is manipulating numbers."

The first person to make the connection between math and music was Pythagoras (of Pythagorean Theorem fame), a philosopher who lived in 5th century BC. For Pythagoras, ratios were everything. He discovered the harmonic progressions in the notes of the scale by finding that musical intervals and pitch of notes correspond to relative lengths of vibrating strings.

Math is at the very heart of music. As drummer and educator Ndugu Chancler once said, "Fractions are the common language for all musicians." And music is a fun and informal way to understand math concepts. Creating rhythm helps in understanding patterns and repetitions; and constructing musical measures helps clarify fractions and equations. For students who are interested in music theory, math is a must.

In MusicMath, students explore the many ways that music and math are connected. They begin by listening and moving to music and constructing simple four-count rhythm patterns. They then extend the patterns to create new arrangements and, eventually, a musical composition. Paper is folded and cut to represent lengths of notes, color-coded to indicate the instrument, and then displayed in measures by hanging on a string or yarn or sticking on a wall. At first students "play" the composition with their bodies (clapping, stomping, snapping), and then they use "found" percussion instruments (pencils, books, blocks). At the museum, they make their own instruments so that they can perform their composition with families and friends at the culminating event.

Equity

Students who don't think of themselves as "good at math," can be great at MusicMath. All students, regardless of gender or ability, have the opportunity to thrive with MusicMath.

Inclusion

Music is an accessible sensory experience for students with and without disabilities. Specific modifications have been included in the activities to promote tactile

NCTM Math Standards

Content Standards

- **Algebra:** Identify and record patterns; extend patterns in multiple iterations; and compare different forms of representation for parts of the pattern.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through solving problems that arise in mathematics and in other contexts.
- **Communication:** Use the language of mathematics to express mathematical ideas precisely.
- **Representation:** Use representations to model and interpret physical, social and mathematical phenomena.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

experiences for students with visual impairments, as well as additional visual cues for students with hearing impairments. Students who are blind or visually impaired can hear the music and feel the notes; students who are deaf or hard of hearing can feel the notes and the beat of the music; all students can create patterns and move to the music.

Cultural Links

Music is an international language; it is an integral and exciting part of many cultures. Begin by asking students to bring in and share the music that relates to their family history. Something interesting to discuss with students is how traditional American music (including blues, jazz, hip hop, rock) originated from diverse cultures.

Career and Role Model Connections

The role models in this theme will inspire students to think about math as a future career. Neil McLachlan, a PhD in applied physics, designs instrumental ensembles and installations. Manjul Bhargava, one of the youngest people to become a full professor, is an expert in number theory and skilled on a small Indian drum called the tabla. Christine Southworth majored in math and creates music by combining technology and creativity. Minerva Cordero and Fern Hunt have pursued their love of mathematics in research and teaching careers. Abraham Nemeth was a mathematician who was born blind. He was the creator of the "Nemeth Code" and "MathSpeak," two inventions that made it possible for people who are blind to study and pursue careers in mathematics. He also taught himself to play piano. Be sure to read these bios and have students complete the related career and role-model activities.

At the Museum

At the museum, students will make their own instruments, using many math skills—measuring, counting, ratios—while having a fun, hands-on experience. They will make drums, tambourines, sound blocks and other percussion instruments using recycled materials such as cans of different sizes and rubber or plastic to make drums, sandpaper for sound blocks, and bottle caps for tambourines. At the culminating event, students form an orchestra to play their compositions on the instruments they have created. Family and friends are the proud audience.

At the culminating event, a joyous noise was heard throughout the museum as students played their original percussion composition on instruments they had created from recycled materials. Fractions were everywhere as whole notes, half-notes, quarter notes, eighth notes, and sixteenth notes rang out. Students led the "orchestra" and played flawlessly. When family members tried to follow the notes, however, it was clear that more practice was needed!

Remember to:

- ☐ **Check out the resources.**
- ☐ **Review the glossary.**
- ☐ **Send out the family letter when you start the theme.**

Dear Families,

Our Afterschool Math Plus theme is called MusicMath. It is a series of hands-on, minds-on activities that will help your child develop essential math skills while having fun.

Math is at the very heart of music, and music is a fun and informal way to understand math concepts. Creating rhythm helps us to understand patterns and repetitions; constructing musical measures helps in understanding fractions and equations. Music is the poetry of math.

In MusicMath, students explore the many ways that music and math are connected. They begin by listening and moving to music and constructing simple rhythm patterns. They then extend the patterns to create new arrangements and, eventually, a musical composition. At first students play the composition with their bodies (clapping, stomping, snapping), and then they use found percussion instruments (pencils, books, blocks). At the museum, they will make their own instruments so that they can perform the music they create for families and friends at the culminating event.

The Plus is a series of career- and role-model activities that help to broaden your child's ideas about who does math, how math is a part of everyday life, and possible math careers.

There are many ways you can help your child make the most of this theme. Share with your child your enjoyment of music and any dances you may know. Talk to your child about the many careers that include math (or go on-line together to find out about them). Ask your child about the composition they will be creating for the family event.

Please feel free to ask a staff person about Afterschool Math Plus and your child's participation.

Sincerely,

Queridas Familias,

El tema de Matemáticas Despues-de-Escuela y MÁS se llama MusicMath (Matematicas-Musica). Es una serie de actividades en las que se usan las manos y la mente, que ayudan a su niño/niña a desarrollar habilidades esenciales de las matemáticas mientras se divierten.

Las matemáticas están en el mismo corazón de la música, y la música es una diversión y una manera informal de entender conceptos de la matemática. Creando ritmo nos ayuda a entender patrones y repeticiones; construyendo medidas musicales que nos ayudan a entender fracciones y ecuaciones. La música es la poesía de las matemáticas.

En MusicMath, los estudiantes exploran las diversas maneras en que la música y las matemáticas están conectadas. Comienzan escuchando y moviéndose con la música y construyendo patrones simples de ritmo. Entonces extienden los patrones para crear nuevos arreglos y, eventualmente, una composición musical. Al empezar, los estudiantes juegan la composición con sus cuerpos (aplaudiendo, pizoteando, sonando sus dedos, etc.) y luego utilizan los instrumentos de percusión encontrados (lápices, libros, bloques). En el museo, harán sus propios instrumentos de modo que puedan realizar la música que crean para el evento, para las familias y amistades.

El MAS es una serie de actividades acerca de carreras y de modelos positivos que ayudan

a engrandecer las ideas de su niño/niña sobre quién participa en las matemáticas, cómo la matemática es parte de la vida diaria, y en posibles carreras dentro del campo de las matemáticas.

Hay muchas maneras en que usted puede ayudar a su niño/niña a aprovechar este tema. Comparta con su niño/niña como usted disfruta de la música y bailes que conoce. Hable con su niño/niña sobre las muchas carreras que incluyen matemáticas (o vaya en-línea juntos a descubrir sobre ellas). Pregúntele a su niño/niña acerca de la composición que ella/el está creando para el evento de familias.

Por favor, siéntase cómodo/a de preguntarle a nuestros empleados sobre el programa

Matemáticas Despues-de-Escuela y MÁS y de la participación de su niña/niño.

Sinceramente,

Afterschool Math Plus

Revised Edition

Theme +4: MusicMath

Who Uses Math? Equity Activity +4: Unexpected Math

Who Uses Math?

Unexpected Math

Math appears in many places—we just don't think of it as math. When we recognize a pattern in tiles on the floor or on the sidewalk, when we estimate how long it will take us to get home from school, when we can predict the beats in the next measure of a song we are listening to, or when we solve problems, we are using math!

1. Ask students to think about when they use math (many students may say they use math in school). Make a list.

2. Ask students to think about the unexpected math in their day. It may be necessary to prompt students since they may not recognize the math in some activities. Include these unexpected math experiences:

    - Using problem solving skills
    - Identifying patterns
    - Measuring recipe proportions
    - Classifying shapes
    - Creating art or craft projects
    - Using symbols to represent an object, amount, or phenomena
    - Keeping score at a sporting event
    - Comparing costs and buying food for a party
    - Playing games or puzzles

3. Students may be surprised at how much math is incorporated into their day and may want to continue to add to the list. Post the group's original list and give students an opportunity to add to it. Revisit this unexpected math list throughout the various activities.

Jessica Scranton for FHI 360.

Afterschool Math Plus

Revised Edition

Theme +4: MusicMath

+1 Creating Rhythm

+1 Creating Rhythm

ASM+ INSTRUCTOR GUIDE

Question
Are music and mathematics related?

Objectives
Students will:

- Look for connections between mathematics and music.
- Identify and replicate rhythms.

Where's the Math?
Students will explore patterns and repetitions in music.

Math Skills Developed
- Identifying math in a new context

Materials
- ☐ Chart paper
- ☐ Markers
- ☐ Paper
- ☐ Pencils
- ☐ Music

Getting Ready
A few days before you begin the MusicMath theme, play various kinds of music as background while students are engaged in their regular afterschool activities. For example, during homework time, choose soft, mellow music; during physical activities, choose something with an upbeat tempo; and for art activities, snack-time, games, etc., play some lively, rhythmic pieces.

Note to Group Leaders:
Make sure the selections are age-appropriate.

As students are engaged in activities, note their responses to the music. Are they swaying, tapping, nodding, or in some way moving to the music? As students work with music in the background, you can pose these questions to individuals:

- Have you heard this kind of music before? Where?
- Do you or your family listen to this type of music?
- How would you move or dance to this music? Fast? Slow?
- Do you know of a particular dance that goes with this music?

Be sure to become familiar with the list of math and music terms that you will be introducing throughout the MusicMath theme (see glossary).

NCTM Math Standards

Content Standards

- **Algebra:** Identify and record patterns; and compare different forms of representation for parts of the pattern.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through solving problems that arise in mathematics and other contexts.
- **Communication:** Use the language of mathematics to express mathematical ideas precisely.
- **Representation**: Use representations to model and interpret physical, social, and mathematical phenomena.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

Canva

Throughout this theme, take note of specific tips for including students who are deaf or hard of hearing in the music activities. As a general guide, it is important to allow students to express their preferences to you about how they would like to "listen." This might involve sitting close to the source of the music, touching the speakers, watching other students to catch the beat, or any other strategy the student may have developed from past experience. In short, never assume what a student can or can't do; give the student the opportunity to share their desires and needs in a safe, encouraging environment.

Part One: Large-Group Discussion and Activity (20 minutes)

1. Conduct a discussion about music with students:
    - What kind of music do you listen to?
    - What do you listen to music on?
    - What is your favorite kind of music, e.g., rock, jazz, folk, hip hop, Latin, classical?
    - What is it about the music that you like?
    - Does your family like the same kind of music? Do you listen together?
    - Does anyone take music lessons? What instrument are you learning to play?

Note to Group Leaders:
You can make a bar graph of "Our Favorite Kind of Music."

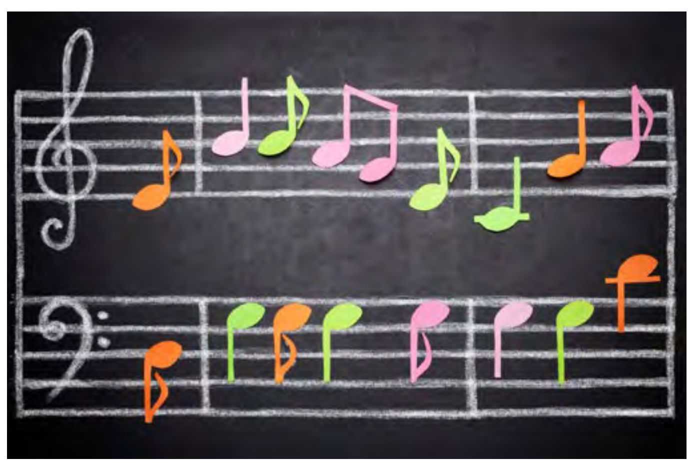
2. Ask students to turn to a partner and pose a question for them to think and talk about:
    - Do you think there are ways that music and math could be related?
3. Write students' ideas on chart paper. Did students mention beats, rhythm, counts, patterns, measures, whole and half notes, etc.? Tell students that during the next few weeks they will be exploring music in many ways and will discover some of the exciting ways that music and mathematics are connected.
4. Hang up the chart and let students know that as they find new ways that link music and math, they will be added to the list.

Note to Group Leaders:
Be sure to return to the chart throughout the various activities and add items to it.

Part Two: Listening to the Music (20 minutes)

Call students together and have everyone listen to a short piece of music (perhaps one of the pieces you've been playing as part of getting ready). You may want to play it twice and ask:

- Can you hear different instruments in the music? Which ones?
- Can you imitate one of the instruments or sounds you hear?
- Does the instrument or sound repeat more than once?
- Is there a beat to the music? Can you clap the beat?
- Is there a rhythm to the music? Can you clap the rhythm?
- Is there a pattern to the music? Can you repeat the pattern?


Canva

For students with learning disabilities, as well as all other students, it might be helpful to have a poster of musical instruments pictures with labels. If the music has lyrics, you might wish to provide written lyrics to students with auditory disabilities.

ASM+
ACTIVITY

Students who are deaf or hard of hearing can participate in the following ways: by sitting in close contact to the speaker or by using lights to show the beat. This can be done by giving one or two (or more!) students flashlights and having them flash on the beat. The room light can also be used to indicate beats.

Consider showing students this segment of *America's Got Talent* where Mandy Harvey, who is deaf, describes her pursuit of music after losing her hearing, or showing her singing a song she created on the show (song begins at 3:25). Encourage students to think about how she was able to "hear" the notes she was singing.

Canva

Part Three: Moving to the Music (30 minutes)

1. Ask students if they dance to music. If so, when? Do they dance to music on their phones, computers, or TV at home? Do they dance in school? Does anyone take dancing lessons?
   - Students with physical disabilities may or may not have experience in dance. Do not assume that they are unable to dance based on your perceptions. Give students with physical disabilities a broad definition of dance—any rhythmic body movement is great!
2. Ask if any students can recommend some music that goes with a dance. If so, ask them to teach everyone. (Students may know line dances or dances that have gone viral.) If not, be prepared with a short dance to music they know. After the dance, discuss the pattern of the steps with students.
3. Ask, "Can anyone think of some ways we used math when we listened to music and danced"? Students may come up with words like counting or pattern. Add them to the chart.

Part Four: Exploring Rhythm (10 minutes)

Play either the piece of music students listened to or moved to. Ask students to listen one time through and then clap the rhythm together.

Again, the use of flashlights to "flash" the rhythms is helpful for students with auditory disabilities.

Part Five: Literacy and Math Identity Activity (10 minutes)

1. Give students time to work in small groups to create poems.

2. After they create their poem, have students practice clapping out the rhythm of the poem.

3. Ask students to perform their poem to the class by reading the words and clapping out the rhythm. Encourage students to teach their poems and rhythms to each other. Explain that we often remember things better when they fit a rhythm.

Additional Literacy Activity

If this activity better suits your students or the goals of your afterschool program, replace Part Five with it. Or add this activity into the math to provide extra math-identity building.

- Visit the library or explore the Internet to look for books that discuss the relationship between music and math. Below are some books you can explore:
  - *Yolanda's Genius* by Carol Fenner, illustrated by Raül Colón (New York: Alladin Paperbacks, 1997).
  - *Functional Melodies: Finding Mathematical Relationships in Music* by Scott Beall (New York: Springer-Verlag, 2001).

Equity

- Be sure to create an environment where students feel comfortable expressing their opinions and ideas—an environment where they can be risk-takers and where they feel their ideas are valued.
- If dancing is an issue for students with disabilities or students who have little experience dancing, think about alternative ways that students can enjoy moving to the music.
- Remind students that Mandy Harvey was deaf when she wrote the song she performed on *America's Got Talent*. How did she do that? What strategies might she have used?

Reflecting on the Activity

- Did I have enough time to do the activity?
- Did I encourage all students to participate?
- Were all students comfortable moving to the music?

Afterschool Math Plus

Revised Edition

Theme +4: MusicMath

+2 Making Rhythm Patterns

+2 Making Rhythm Patterns

Question
Are there patterns in music rhythms?

Objectives
Students will:

- Create a four-count pattern.
- Replicate the pattern.
- Represent the pattern.
- Perform the pattern as represented.
- Explore the use of notation to make music replicable.

Where's the Math?
Music uses mathematics in its design and implementation. In this activity, students will design a musical pattern, represent the pattern and then play the pattern using the representation.

Math Skills Developed
- Understanding a four-count pattern
- Being able to represent a pattern
- Identifying variables that can be used to describe the pattern

Materials
- ☐ Squares of paper or cardboard (8" X 8") to indicate each part of the four-count pattern
- ☐ Markers
- ☐ Glue sticks
- ☐ Wax-covered yarn or pipe cleaners
- ☐ Paper to accommodate four of the squares

Getting Ready
Gather all the materials. Practice clapping or tapping out a four-count pattern. (This can be done by clapping four times and thinking the word "blue" in between claps.) Be sure to take your time as you count out the pattern. Review the glossary for music and math words.

Each pattern needs to have four-counts; the arrangement of those four-counts can vary, but once recorded as a representation, they must be played as represented.

NCTM Math Standards

Content Standards

- **Algebra:** Represent, analyze, and generalize a variety of patterns with words and symbols; relate and compare different forms of representation for a relationship; and develop an initial conceptual understanding of different uses of variables.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through solving problems that arise in mathematics and other contexts.
- **Communication:** Use the language of mathematics to express mathematical ideas precisely.
- **Representation**: Use representations to model and interpret physical, social, and mathematical phenomena.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

Canva

Part One: Large-Group Activity (10 minutes): Connecting with *Stomp*
One of the ways MusicMath will be most engaging for students is by encouraging them to make connections with contemporary music. A great inspiration is the sensational musical, *Stomp*. *Stomp* is a high-energy, percussive symphony, coupled with dance, played entirely on unconventional instruments, such as garbage can lids, buckets, brooms, and sticks. It is a movement of bodies, objects, sounds—even abstract ideas, made completely original by using everyday objects in totally nontraditional ways. You can listen to audio and video clips on the Stomp Online YouTube channel (https://www.youtube.com/@StompOnline1994) or Stomp Online, the show's official website.

ASM+
ACTIVITY

Part Two: Small-Group Activity (20 minutes): Making Rhythm Patterns

1. Divide students into groups of four and tell them that they are now in their "music groups." Give each group four 8"x 8" squares of paper, markers or crayons, a glue stick, and a large piece of paper to accommodate the four squares.
    - If students with visual impairments are present, consider using cardboard rather than paper for the four squares. This will provide a raised tactile cue. Also, provide wax-covered yarn or pipe cleaners, in addition to markers or crayons, for the pattern they design.
2. Ask each group to create a four-count rhythm pattern, e.g., 2 claps, 2 stomps; or 1 stomp, 2 finger snaps, 1 clap
3. Ask each group to represent each of their counts on one of the squares of paper. They can use words, pictures, or symbols, but someone else must be able to understand and repeat the pattern.
    - If students with hearing impairments are present, encourage their teams to create a silent pattern; that is, rhythmic actions that do not involve sound, such as winks, waves, bows, etc. This reinforces the fact that rhythm and patterns do not have to be heard to be communicated. Moreover, it involves students who are deaf or hard of hearing in a meaningful way.
4. Have students glue down the counts, in the order of their pattern, on the large piece of paper.

Canva

Part Three: Large-Group Activity and Discussion (20 minutes)

1. Ask each group to perform its pattern and explain how they represented it.
2. After everyone has had a chance to perform, talk about the rhythm patterns. How were they the same? How were they different? Was tempo (speed of the rhythms) one of the major differences? How (faster, slower)? Did the sounds seem like music?

3. Ask students what they would need to do so that others could play their rhythm patterns. Point out that it would be easier to "read" the music if the same symbols were used throughout (like notes on a sheet of music). Then, decide as a group on symbols to represent each action. Depending on the age of the students, they might want to use a hand or a triangle for clapping, a foot or a circle for stomping, fingers or a square for snapping, etc.

4. Hand out four more 8"x8" squares of paper to each group so that they can rewrite their patterns using the agreed-upon symbols and then glue them onto a large piece of paper. As above, provide cardboard and wax-covered yarn or pipe cleaners so that students with visual impairments can create a tactile pattern.

5. Ask students to write the names of the people in their group on their rhythm pattern and then exchange and play each other's music.

Part Four: Literacy and Math Identity Activity (15 minutes)

1. Ask each student to write a short song or rap and have an open mic session for students to perform.

2. Encourage students to create lyrics that rhyme, use repeating words, or have a pattern.

Additional Literacy and Math Identity Activities

If this activity better suits your students or the goals of your afterschool program, replace Part Four with it. Or add this activity into the math to provide extra math-identity building.

- Ask students to share their favorite song. Do the words sound like poetry? Do students see a pattern?
- Find a copy of the song "Twinkle, Twinkle Little Star" by David Cedeño and His Orquestra for an example of a very popular children's song put to a Latin beat. Share with students that the original composer was Mozart.

Equity

Encourage groups to listen respectfully to each other as they play their music patterns and offer constructive feedback. Be sure that everyone's ideas are heard. Create mixed groups of students for this activity.

Reflecting on the Activity

- Were students able to work in cooperative groups to create their rhythm patterns?
- Did everyone understand the concept of representation?
- Did all students participate in performing the rhythm patterns?

Afterschool Math Plus

Revised Edition

Theme +4: MusicMath

+3 Composing Music

+3 Composing Music

Question
How can we arrange a series of sounds to create a new piece of music?

Objectives
Students will:

- Understand that each measure of music organizes a group of beats.
- Experiment with repeating measures (e.g., four snaps repeated four times) and contrasting measures (e.g., four stomps repeated four times).
- Identify and extend patterns to create new arrangements of their sounds.

NCTM Math Standards

Content Standards

- **Algebra:** Make generalizations about patterns; express mathematical relationships using representations for the pieces of the pattern; and relate and compare the properties of the representation.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through solving problems that arise in mathematics and other contexts.
- **Communication:** Use the language of mathematics to express mathematical ideas precisely.
- **Representation**: Use representations to model and interpret physical, social, and mathematical phenomena.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.

Where's the Math?
Musical compositions include a variety of patterns and repetitions. Figuring out what sounds most pleasing involves creating patterns, deciding which ones are most pleasing, and then adding variations to those patterns. In this process, students learn about arithmetic progressions and experiment with combinations and permutations. Combinations and permutations are ways of creating variety while still using the same elements. For example, students can create combinations by mixing up the notes and sounds in the measure they create in this activity. They can create permutations in their music by using one measure (that doesn't change) to introduce another measure to create an ordered pattern (AB, AC, AD, etc.).

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Math Skills Developed
- Identifying parts of a pattern
- Integrating parts of one pattern with parts of another pattern
- Representing the newly formed pattern that results from this integration
- Developing a sequence

Materials
- ☐ Copies of four-count measures previously developed
- ☐ Paper or cardboard (8" x 8" squares) for additional representations of the new patterns
- ☐ Scissors
- ☐ Glue or tape
- ☐ Chart paper
- ☐ Pencils and markers
- ☐ Wax-covered yarn or pipe cleaners
- ☐ Construction paper
- ☐ Sticky notes in different colors (optional)

Getting Ready
Create two four-count measures, one using the symbols for four claps and one using the symbols for four stomps, and mount on construction paper. It may be easier to use two different colored pieces of construction paper so that the students can easily see which measure the "beats" came from once the measures are cut. This may make the permutations easier to understand.

Note to Group Leaders:
This activity may need to be conducted over several days. Remind students not to rush as they play their patterns.

Part One: Large-Group Composition

Remind students of the four-count rhythm patterns they created in the last activity. Explain that they are now going to arrange sounds to create a new piece of music.

1. Show students the sheet of construction paper with one of the four-count measures that you prepared for this activity (e.g., the four claps). Ask students to clap the measure in unison. Now have them repeat the measure a second time. Explain that they now have an eight-count piece of music. With construction paper, have students help you make a representation of what the eight-count piece looks like. Draw a line between each measure.
2. Ask students to repeat the new eight-count piece of music twice. What does it sound like? Guide students from one symbol to the next in a steady beat (e.g., point, tap your foot). Ask students what words they would use to describe this piece. Note their words on a piece of chart paper, under the title, "Version One."
3. If there is time, have students repeat the process with the other four-count measure (e.g., four stomps).
4. Explain that now students are going to create two new measures using two elements from the first measure and two elements from the second. For example: If the first measure was four claps and the second measure was four stomps, the new measures will have two claps and two stomps and then two claps and two stomps. Record these new measures using the original symbols.

Note to Group Leaders:
You may want to cut up the previous measures and tape them to a new piece of paper to show that you are using the existing measures to create a new measure. Again, you will have an eight-count composition consisting of two measures. You can also use different colored squares for each original measure so that students can trace back the origin of the beats.

5. Have students repeat this new eight-count piece several times. Ask them to describe this new composition. How does this piece of music sound? Note their comments on the chart paper under the title, "Version Two."
6. Ask, "If we want to create another composition, can anyone guess what we might do now?" Gather ideas from the students.

Note to Group Leaders:
If students need help in getting started, you could point out: In the first composition we had four sounds that were repeated. In the second, we took two of the sounds and combined them with two of the sounds from another piece of music. Do you see a pattern here? What do you think we will do next?

7. Once students have recognized the 4, 2, pattern, ask them what would follow if the pattern continued: 4 divided in half is 2; 2 divided in half is 1. Have students cut the previous pattern into single sounds and place them on a sheet so that one sound is followed by a different sound. There will be eight sounds. Have students construct two new measures. Remind them that each measure has four sounds. Again, use sticky notes in different colors rather than pasting as it would eliminate the use of scissors, which may be difficult for students with motor skills challenges.
8. Repeat this new eight-count piece several times. Ask students to describe this new composition. How does this piece of music sound? Note student comments on the chart paper under the title, "Version Three."

Part Two: Small-Group Composition

1. Have students get into their four-person music groups and give each group its original four-count rhythm pattern (which they can now call a measure). Ask each group to use its measure to create an eight-count piece of music (two measures). Then, going through the steps you just did in the large group, ask each music group to come up with three variations on their eight-count pattern.

2. Have each music group perform its eight-count compositions. If time permits or on another day, ask each group to perform the version that it liked best for everyone. Ask students to describe their compositions and write down their ideas on chart paper.

Note to Group Leaders:
Older students may want to explore using all three of their versions in various combinations. They may be interested to know that musical compositions include a statement, contrast, and return. This process is a permutation. Can they construct a piece of music that uses these elements with the three versions they have now created?

Part Three: Literacy and Math Identity Activity (30 minutes)

1. Have students create their own short story by using sound effects or musical compositions to describe their story (beginning, middle, and end).

2. Ask students to share their stories in a small group or the whole group.

Additional Literacy Activity

If this activity better suits your students or the goals of your afterschool program, replace Part Three with it. Or add this activity into the math to provide extra math-identity building.

- Read Robert Louis Stevenson's "Bed in Summer" with students. Clap out the rhythm. Can students identify the "measures"? Ask students where they think a measure begins and ends.

Equity

Remember to refer to the small groups as "music groups" and not "teams" to reduce competition. Include mixed groups and be sure that every member of the group has a chance to participate.

Reflecting on the Activity

- Did I allow enough time for the activity?
- Did students understand the sequencing of a pattern?
- Did I make sure that all students had a chance to perform?

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Afterschool Math Plus

Revised Edition

Theme +4: MusicMath

+4 Discovering Music Fractions

+4 Discovering Music Fractions

ASM+ INSTRUCTOR GUIDE

Question
How can we create a more complicated piece of music and represent it so that others can play it?

Objectives
Students will:

- Identify fractional parts.
- Use representation to denote specific fractions (e.g., different sizes of paper).
- Record values using fractional notation.
- Understand the concepts of combinations and permutations.

Where's the Math?
Using a model of a whole for each measure, students will explore ways to create different musical compositions using four-count patterns. Students will see that the whole of the measure can be represented by a large square (8″ x 8″); from that whole, they will create fractions—four-counts or 1/4 of the whole, a smaller square (4″ x 4″). They will also create models for 1/8 or 1/16 of the whole. Students will modify the model, and experiment with a variety of constructions for four-count patterns.

NCTM Math Standards

Content Standards

- **Number Operations:** Construct models for fractions; label the fractional parts; construct arrangements of fractional parts that equal a whole; and experiment with using equivalents for different arrangements.
- **Algebra:** Represent a variety of patterns using tables and/or symbols; relate and compare different forms of representation; and explain the changes in patterns based on equivalent values.

Process Standards

- **Communication:** Organize and consolidate mathematical thinking through communication; communicate mathematical thinking coherently and clearly to peers, teachers and others; and use the language of mathematics to express mathematical ideas.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.
- **Representation:** Create and use representations to model and interpret physical phenomena.

Math Skills Developed
- Understanding parts of the whole
- Becoming familiar with fractional notation
- Using fractions to create a whole
- Understanding combinations and permutations

Materials
- ☐ 8-inch squares of paper in various colors
- ☐ Scissors
- ☐ Glue or glue sticks
- ☐ String or yarn
- ☐ Clothespins or paper clips

Handouts
- ☐ Notes Values Chart
- ☐ Music Square Template

Getting Ready
Make copies of the Note Values Chart handout for each student. If possible, enlarge a copy onto a large sheet of chart paper. Make copies of the Music Square Template handout so that each student will have several copies. Hang string or yarn across the room to hang the music as a visual composition. Be sure to hang the string or yarn at a height accessible for students who use wheelchairs.

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Part One: Large-Group Activity: Demonstration (20 minutes)

Introduce students to the activity. You could say: "In our previous activity we looked at ways to create compositions using rhythm patterns based on variations of two measures that used two different sounds. We combined these two sounds using a pattern that divided up those sounds in halves. In this activity we are going to look at another way to create compositions based on the length of time that we allow for each sound. Today we are going to find a way to indicate how long we want our sound to last. Also, we need to remember that in this activity, our squares are not just worth one beat; they are worth four beats, an entire measure! We will see how we can divide that measure into different beats to create different music patterns."

1. Remind students that they have been creating four-count measures and that each count was represented by a sound that was of equal duration. Ask students to think about the music they like to listen to and point out how, in that music, some sounds are longer and some shorter.

2. Take a piece of 8"-square paper and explain that it represents a four-count measure. Ask for a volunteer to help you fold the paper in half, and half again, so that it's divided into four equal smaller squares.

3. Then cut the squares apart and hang them on a piece of string or yarn using clothespins or paper clips. Point out that it is important to indicate where a measure begins and where a measure ends. Ask for another volunteer to hang a piece of yarn to show the beginning and end of the measure.
   - Note: Once you have one measure identified, you don't need to indicate the beginning of the next measure—it will begin where the other measure has ended.
   - For students who are blind, pre-fold three sheets of paper; one in quarters, one in eighths, and one in sixteenths. Allow the students to feel the divisions of the paper.

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4. Each square represents one of the sounds in the four-count measure. Ask students: "If we know that the four-count measure is a whole, what part of the whole does each of these small squares represent? How can we write that?" Help students understand that they can write the mathematical equation 1 = ¼ + ¼ + ¼ + ¼. Ask, "Is this equation true?"

5. Ask students how they could indicate that they want a sound (e.g., a clap) to be two quick sounds (two quick claps). They could divide the square into two pieces. Ask, "If we divide one of these small squares into two pieces what will each of these pieces represent?" Explain that one way to think about this is by figuring out how many of these small rectangles it will take to cover the original large square. It will take eight—so each of these pieces will be labeled ⅛. For older students, you could also say that ¼ divided by 2 = ⅛.

6. Ask students to try to fold and cut a piece that represents ⅛ into two pieces. Each piece will look like a very small square. What part of the whole does it represent? Students will be able to see how many of these it will take to cover the whole—the large square—or they can perform a division operation using fractions: ⅛ divided by 2 = 1⁄16.

7. Now students have a way to show ¼, ⅛, and 1⁄16 sounds. But how would they know what sound to use? Suggest a color code, e.g., red indicates a clap, blue indicates a stomp, etc. Let students decide on the code and place it in a prominent place so that students can refer to it as they work in their small groups. For students who are colorblind, use a clearly visible symbol in black for easier identification.

Part Two: Discovering Music Fractions (20 minutes)

A great resource to help students relate fractions to music is "Drumming in Fractions," a short video by drummer and educator Ndugu Chancler. At age 6, Chancler made his own drums out of coffee cans. He once said, "Fractions are the common language for all musicians." He demonstrates, with great drumming, how quarter, eighth, and sixteenth notes are used to express rock or funk, jazz, swing, and hip hop.

1. Play the video for students.

2. Then hand out copies of the Note Values Chart. Refer to the fraction graphic on the chart and discuss how the lengths of notes are like fractions: for example, two half notes are equal to (last as long as) a whole note; four eighth notes are equal to (are the same length as) one half note; and so on.

    - You could say that a ¼ note is a regular clap; a ⅛ note indicates a shorter time for a clap; and a 1⁄16 note an even shorter time. These may be hard to imagine, but practice will help. The time taken to make two ⅛ claps would equal the time for one ¼ clap. Tell students that they will be working in their music groups to try this out and to see if they can find a way to measure their claps or stomps or snaps. Remind them that they also have a color code (or symbol) for each of the sounds.

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Part Three: Small-Group Activity (20 minutes)

1. Have students get back into their four-person music groups and give them the following instructions: "In your music groups, see if you can find new ways to construct your four-count measure. Remember that the total for each of your measures will be equal to the large square. You can see if you have an accurate model of a four-count measure by checking that the various pieces you used in your measure equals the large square."

2. Have each group create a measure that uses a variety of different notes (both sounds & lengths). Students can use their original measure or create a new one. Once the groups decide what sounds they want to use in their measure, they will need paper in the colors that correspond to those sounds.

- For students with physical disabilities, snapping, clapping, or stomping may not be feasible. Offer alternatives that all students in the group can do (some possibilities are tongue-clicking, humming, finger-tapping, ringing a bell, etc.).

3. Have each group experiment with a variety of notes for one sound. What does a ¼ note clap sound like? What does a ⅛ note clap sound like? What does a 1⁄16 note clap sound like? Have them make combinations to create a measure. Start out simply with just ¼ and ⅛ notes. Ask:
   - What combinations can you use for a measure?
   - Is there one combination that you like better?
   - Can you build another measure that uses these notes in another combination?
   - What happens if you introduce a 1⁄16 note?

Note to Group Leaders:
Be sure that students remember that the total needs to add up to a whole (1). If you want to add 1⁄16 note, you need to take something away. For example, if you take away a ¼ note you could add ⅛ note and two 1⁄16 notes. You might want to ask them to write a fraction sentence that proves that each measure is a whole. For example, 4 eighths + 1 half = 1 whole.

4. For younger children, consider limiting the kinds of sounds so that they focus on making the sounds different lengths (¼, ⅛, etc.) as variations. This will also increase the amount of math they do as they compose.
5. Have each music group agree on two measures they would like to present to the whole group. Give them time to practice.

Part Four: Large-Group Activity (20 minutes)

1. Ask each group to volunteer to perform their two measures. Allow time for each group to have an opportunity to explain how they developed their measures and why they used the notes that they did.
2. Ask the rest of the class to see if each measure equals a whole (1).
3. Have each group hang up their measures on the yarn or string.
4. Remind students to use yarn to indicate when a measure stops and the next one begins.

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Part Five: Literacy and Math Identity Activity (10 minutes)

Read the following quote by Leon Battista Alberti, a famous mathematician and artist from the 15th century, to the students: “We shall therefore borrow all of our Rules for Proportions from the musicians, who are the greatest masters of this sort of number, and from those things wherein nature shows herself most excellent and complete.” Ask students what they think Alberti meant by this. Can they give an example?

Additional Literacy and Math Identity Activities

If this activity better suits your students or the goals of your afterschool program, replace Part Five with it. Or add this activity into the math to provide extra math-identity building.

- Play the music from Prokofiev's Peter and the Wolf. Ask students to write a story to go with the music. Play the music again as they are writing. After they have written a story, play the introduction (with the assignment of animals to the instruments) for the students. Do they hear the story? Are they surprised?
- Go to the library and find a copy of *Music Math: Exploring Different Interpretations of Fractions* by Kathleen Collens (New York: PowerKids Press, 2004).
- Share resources about Ndugu Chancler, and ask students what they can learn from him about math and a music career. Suggested resources include:
  - "Remembering Ndugu" from the University of Southern California (USC) Thornton School of Music by Evan Henerson (Feb. 8, 2018).
  - "Ndugu Chancler Interview - Drummer's Resource Podcast 009" from Drummer's Resource podcast (Jan. 16, 2014).
  - "Leon 'Ndugu' Chancler" from Percussive Arts Society's Hall of Fame by Rick Mattingly and their short video recognizing Chancler's induction to their Hall of Fame.

Equity

As this activity develops, some natural leaders may evolve—students who can read music or play an instrument. Try to include their expertise, but be mindful of giving a voice to students who may not have had previous musical training.

Reflecting on the Activity

- Did I help students understand the relation between the length of musical notes and fractions?
- Do I need to schedule more time for students to practice with fractions?
- Was the room large enough for the visual composition?

Afterschool Math Plus

Revised Edition

Theme +4: MusicMath

+5 Orchestration

+5 Orchestration

ASM+
INSTRUCTOR GUIDE

Question

What is involved in conducting an orchestra?

Objectives

Students will:

- Perform their musical composition as a group.
- Use found percussion instruments.
- Conduct an orchestra.

Where's the Math?

Students will bring the understanding of fractions gained in the previous activity to the task of conducting and performing a musical composition. They will recognize that a whole note can be divided into halves, quarters, eighths, and sixteenths. Students will perform to a beat, which will require being able to keep a rhythm and come into the composition at different times. Students will be challenged when asked to extend the pattern.

Math Skills Developed

- Counting and keeping time to a beat
- Following a pattern
- Understanding parts of a whole
- Following directions
- Understanding the value of practice

Materials

- ☐ Visual composition from previous activity
- ☐ A piece of sheet music
- ☐ A short pointer, or long pencil—to act as a baton
- ☐ Metronome (optional)

Getting Ready

Be sure to conduct this activity in a place and at a time that your students can move around and be noisy. Bring in a piece of music so students can see how notes are used as symbols.

Metronomes are electronic devices that tap out beats. Some also have visual representations (flashing lights or moving arms). Metronomes can be extremely helpful in allowing students who are deaf or hard of hearing "see" the beat. They also allow students with visual impairments to adjust beats in an easy and auditory way. Metronomes are available online or at local music stores; you also can search for a metronome app.

NCTM Math Standards

Content Standards

- **Number Operations:** Develop and use strategies to estimate results and judge reasonableness; develop, analyze, and explain methods for solving problems involving proportions.
- **Algebra:** Represent, analyze and generalize a variety of patterns with words and symbols; relate and compare different forms of representation for a relationship.

Process Standards

- **Problem Solving:** Build new mathematical knowledge through solving problems that arise in other contexts; apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematical problem solving.
- **Communication:** Organize and consolidate mathematical thinking through communication; communicate mathematical thinking coherently and clearly to peers, teachers, and others; use the language of mathematics to express mathematical ideas precisely.
- **Connections:** Recognize and apply mathematics in contexts outside of mathematics.
- **Representation:** Create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems.

Canva

ASM+
ACTIVITY

Note to Group Leaders:
This activity will take practice and may need to stretch over several days. It will become the culminating activity for the theme.

Part One: Large-Group Discussion: Creating and Conducting an Orchestra

1. Ask the students if they have ever attended a musical performance that involved a conductor. You may need to remind them that if they have music class, the music teacher will sometimes act as a conductor. Give students time to look at the piece of music you brought in. Do they recognize the different lengths of notes? Show students the Braille music sheet to show how music is represented for people who are blind.

2. Ask for a volunteer to be the conductor to keep the beat while the class performs several measures of the "fraction music" that they previously composed. Can anyone describe a beat? Point out that a beat is a regular pulsation in the music that you can tap your foot to. It is the pulsation that a conductor or band leader typically indicates by waiving a baton.

3. Ask the students to review their music. Do they want to add more kinds of sounds? Notes of different lengths? Give students time to make changes to their music.

4. Let students take turns conducting as they try out their new composition.

   - Notes on Conducting: All students should be facing the conductor, who is standing facing them. The conductor can keep the beat by clapping or by beating on a table (or in the air) with a baton (a short pointer or long pencil can be used). Point out how much easier it is to follow the conductor if the beat is steady. If students who are blind or have low vision are present, have the conductor tap out the beat on a table so that it is audible.

Part Two: Performing with Your Body

1. First, have students perform their visual composition by using their bodies (clapping, stomping, etc.) to play the music. Have students take turns being the conductor.

2. Ask students to divide into sections of an orchestra—for example, the clapping section, the stomping section, the clicking section. Have another volunteer conductor keep the beat while the class performs as an orchestra.

Note to Group Leaders:
Coming in on the beat and staying together as an orchestra takes lots of practice. Be sure to encourage students and provide lots of positive reinforcement.

Part Three: Found Percussion

1. Have students look around the room and find things that can be used as percussion instruments such as pencils, books, pots, lids, blocks, etc.

Note to Group Leaders:
This would be a great time to revisit the inspirational short videos from Stomp, where garbage can lids, buckets, brooms, and sticks become wonderful percussion instruments.

2. Have students color code each "instrument" and then perform their composition using their "found" percussion. Shapes or symbols rather than colors should be used to "code" the instruments if students who are colorblind are present.

Part Four: Literacy and Math Identity Activity (10 minutes)

Some mathematicians refer to certain kinds of mathematics as "elegant." Classical music is frequently referred to as elegant. Ask students to think about what "elegant" means. Ask them to give examples explaining how math and music can be elegant.

Additional Literacy and Math Identity Activities

If this activity better suits your students or the goals of your afterschool program, replace Part Four with it. Or add this activity into the math to provide extra math-identity building.

- Some musicians use math to make their music different. Allow students to complete a short research project about one or more of these musicians, describing how they use math to create their compositions.

Equity

Encourage students to enjoy the variety of sounds and music that they produce. What sounds like music to some students may seem like interesting noise to others! Encourage students to be respectful of each other's compositions and contributions.

Reflecting on the Activity

- Did students have enough time to practice?
- Did I encourage students in their efforts?
- Did I make sure that all students participated?

Afterschool Math Plus

Revised Edition

Theme +4: MusicMath

+6 Career and Role Model Connections

Career and Role Model Connections

Manjul Bhargava, Neil McLachlan, and Christine Southworth have found creative ways to combine their interests in music and mathematics. Fern Hunt and Minerva Cordero have been interested in mathematics as a career from early on. Minerva Cordero learned to love learning and teaching from her mother who valued education above all else. Abraham Nemeth, who was born blind, loved to figure out numbers as a child and became a mathematician.

1. Manjul Bhargava is a professor of mathematics with a special area of expertise in number theory. He also plays a small Indian drum called a tabla.

2. Minerva Cordero enjoys research and teaching. In 1995, she received the Professor of the Year Award given by the student chapter of The Mathematical Association of America at Texas Tech University.

3. Fern Hunt uses both computer model and analytical methods in her work. She also encourages women and minority students to pursue mathematics.

4. Neil McLachlan combines his knowledge of physics with his love of music. He is a composer and designer of tuned percussion instruments.

5. Abraham Nemeth invented the "Nemeth Code" and "MathSpeak" so that people who are blind, as he was, can become mathematicians. He taught himself to play piano.

6. Christine Southworth is an expert on the tuning of a traditional Indonesian ensemble of percussion instruments called a gamelan. She is also the director of Ensemble Robot, which creates music by combining technology and creativity.

Make copies of the biographical information for students. Ask for volunteers to read the biographies aloud.

- Ask students to brainstorm a list of careers that involve math and music. Chart their responses.
- Borrow a copy of *101 Careers in Mathematics* by Andrew Sterrett (Washington, DC: Mathematical Association of America, 2002) from the library to share with students. Share the list of math careers with students—are some of them surprising?
- Ask students, "What is a role model?" Chart a list of characteristics of a role model. Ask students if they are ever role models for younger students.
- Search the Internet for information about Ray Charles and Stevie Wonder—both African American musical geniuses who are blind. Have a discussion about how people who are blind read music and play instruments.
- Invite a musician (professional, parent, sibling, friend) to the afterschool program. Ask them to bring an instrument to play for the students. Tell the musician that the students are looking at connections between music and math: Can the musician think of some examples? How have they used math as a musician?
- Ask students to interview their parents. How do they use math in their lives? Give students some examples of careers that use math that are not the usual math careers: musicians, carpenters, nurses, teachers, cashiers, mechanics, etc.

Afterschool Math Plus

Revised Edition

Theme +4: MusicMath

+7 Handouts

MusicMath Role Model Biographies

ASM+
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Manjul Bhargava
An Artist in Music and Math

Manjul Bhargava is a professor of mathematics at Princeton University and was one of the youngest people to ever become a full professor. He is also an expert in number theory and an expert on a small Indian drum called the tabla. The tabla is a hand drum that is used to make music with rhythmic patterns.

When Bhargava was three years old, he heard his mother playing the drum and wanted to do it, too. His mother teaches mathematics and plays the tabla.

Manjul Bhargava studies numbers that describe things in nature. He says the same kinds of patterns and number sequences appear in the rhythms of classical Indian music. He feels both math and music create beauty and elegance.

Read more about Manjul Bhargava.

Minerva Cordero
Researcher and Teacher

Minerva Cordero's mother valued education and often said to her children, "The best thing I can give you is an education." Minerva Cordero and her five siblings could be found every night doing homework and talking about what they learned in school.

While Cordero was in graduate school at the University of Puerto Rico, she was awarded a National Science Foundation Minority Graduate fellowship. She continued her graduate studies at the University of Iowa and received her Ph.D. degree in 1989.

Cordero enjoys research and teaching. In 1995, she received the Professor of the Year Award given by the student chapter of The Mathematical Association of America at Texas Tech University. She is a professor of mathematics at the University of Texas-Arlington.

Fern Hunt
Encourage Mathematics

Fern Hunt had decided by the time she was 15 that she wanted to study mathematics. She received both her M.S. and Ph.D. degrees from the Courant Institute of Mathematical Sciences at New York University.

To encourage students, Hunt suggests that they should really take the time to find out who they are and what they like. She challenges them to find their own path and not to follow along with what everyone else is doing.

Fern Hunt has wide research interests and uses both computer models and analytical methods in her work.

She has been a participant in the Conference of African American Researchers in the Mathematical Sciences and also taught in a summer program to encourage women and minority students to pursue graduate study in mathematics.

Neil McLachlan
Using Math to Make Music

Neil McLachlan combines his knowledge of physics with his love of music. After getting his Ph.D. in applied physics, he worked with dance and theatre companies as a composer and sound designer. He has explored new tuning systems and designs instrumental ensembles and installations.

McLachlan has written music for international and national dance and theatre companies and, in 1989, he created the physical-music theatre company called GongHouse. He designed and constructed tuned percussion ensembles, including a set of 30 cast-bronze gongs.

McLachlan has worked with the faculties of music, psychology, and engineering at the University of Melbourne, Australia. He designed a chamber orchestra of tuned percussion instruments that can be used in schools and music therapy.

Abraham Nemeth
Mathematician and Inventor

Abraham Nemeth was born in 1918 on the Lower East Side of Manhattan. He was blind at birth. Nemeth came from a large and loving family of Jewish, Hungarian immigrants whose first language was Yiddish. As a child, his father took him on neighborhood walks, letting him touch the raised letters on mailboxes and license plates and orienting him to directions by saying things like, “Right now, we are walking west, as soon as we turn the corner, we will be walking south.” Nemeth’s mother let him walk to the corner store when he was a boy of six or seven. She would give him money and a list of groceries to bring back. Of course, he didn’t have to cross any streets, and the store was owned by his grandfather. Nemeth said that the task of figuring out the costs of things in his head was good practice for his later career.

When it came time to go to school, Nemeth attended the local public schools where he spent most of the day in a classroom with students who were not blind. He did have special training in a resource room where he was taught to read Braille and use a typewriter. He taught himself to play the piano using Braille music books. Nemeth also attended a New York City public high school and Brooklyn College.

Nemeth was always interested in mathematics. Because of his training at home and in school as a child, he was able to write numbers and words on the blackboard. However, Nemeth knew that he needed a way to notate the mathematical concepts that were essential to his teaching in Braille. Since no Braille system for mathematics was available, he invented his own form of Braille notation, which became known as “the Nemeth Code.” The Nemeth Code, invented in 1952, is still in wide use today. He also invented MathSpeak, a system for orally communicating mathematical text.

Nemeth went on to have a distinguished 30-year career as a mathematics professor at the University of Detroit. In the 1960s he studied computer science and created the University’s program in that subject. Nemeth retired from teaching mathematics in 1985 and died in 2013.

Christine Southworth
A Mathematician Who Creates Music

Christine Southworth majored in math at the Massachusetts Institute of Technology (MIT) and received her B.S. in math and music in 2002. She wrote her senior math thesis on the tuning of the gamelan (a traditional Indonesian instrumental ensemble of percussion instruments). She is the director of Ensemble Robot, a musical ensemble, and she creates music by combining technology and creativity.

Southworth is now studying computer music and multimedia composition at Brown University. She writes for different instruments—Western ensembles, Balinese gamelan, electronic music, and music for robots!

Her compositions have been played by various individuals and groups, including Gamelan Galak Tika, Ethel, Bang on a Can, Arnold Dreyblatt’s Orchestra of Excited Strings, Alarm Will Sound, the NEC Wind Ensemble, and Ensemble Robot. Her music is played at the Boston Museum of Science, Mass MoCA, Jordan Hall, MIT, Wesleyan University, and in Bali, Indonesia.

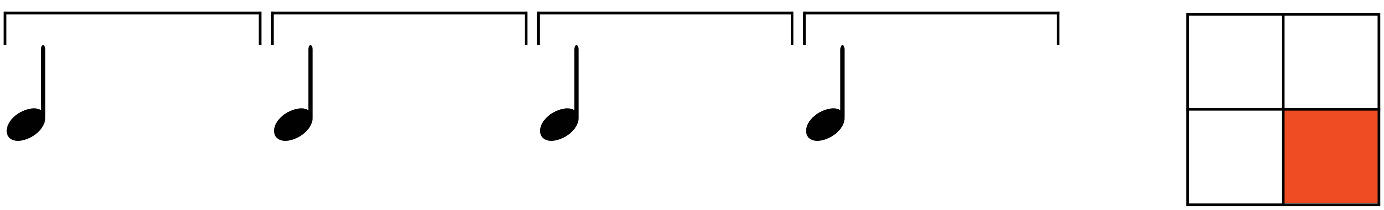
Notes Value Chart

Brackets show how many of each value are required to fill the same amount of time. For example, a whole note equals two half notes or four quarter notes.

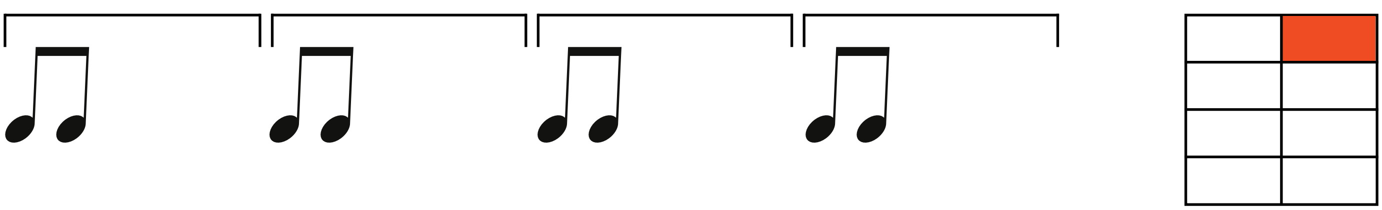
Whole Note:

Half Notes:

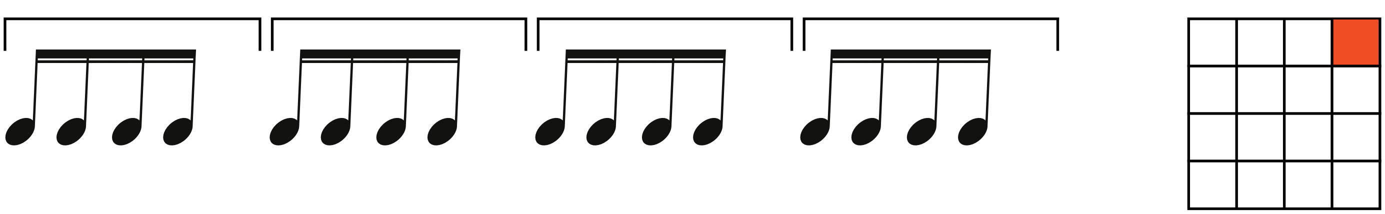
Quarter Notes:



Eighth Notes:



Sixteenth Notes:



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Music Square Template

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Theme +4: MusicMath

+8 Resources

Books

101 Careers in Mathematics by Andrew Sterrett (Washington, DC: Mathematical Association of America, 2002). Each of the jobs profile real people in real jobs and includes information on how they prepared for their careers.

African American Musicians by Eleanora E. Tate, edited by James Haskins (New Jersey: John Wiley & Sons Inc., 2000). Presents biographical profiles of African Americans, both legendary and less well-known, who have made significant contributions to music in the United States over the past 200 years.

Functional Melodies: Finding Mathematical Relationships in Music by Scott Beall (New York: Springer-Verlag, 2001). Capitalize on students' natural interest in music to deepen their understanding of math. With the engaging activities and the easy-to-use music CD, you can help students connect musical ideas to mathematical concepts.

Music Math: Exploring Different Interpretations of Fractions by Kathleen Collens (New York: PowerKids Press, 2004). This book explains math fractions by means of musical notation.

Yolanda's Genius by Carol Fenner, illustrated by Raül Colón (New York: Alladin Paperbacks, 1997). The popular 1994 Newbery Honor book is the inspiring story of a strong ten-year-old girl who looks after her troubled younger brother, trying to help the adults in their life recognize his amazing musical talent.

Websites

Create a Graph
This website provides the tools to create different kinds of graphs (line, bar, area and pie) that can be printed when completed. A great resource for older students.

Drumming in Fractions - The Futures Channel
An inspiring and short video (3:02 minutes) on drummer and educator Ndugu Chancler demonstrating drumming in fractions.

Electric Slide
A description of one version of the Electric Slide.

International Alliance for Women in Music (IAWM)
A global network of women and men working to increase opportunities and promote the music of women.

Mandy Harvey on America's Got Talent
A short clip of Mandy Harvey, a musician who lost her hearing at age 18, performing her original song on *America's Got Talent* (starting at 3:25).

Math Rock
Learn about a style of rock music that emerged in the late 1980s.

Songs for Teaching
Sound clips and lyrics for hundreds of songs that cover many content areas including math and science; organized by subject and age group.

Afterschool Math Plus

Revised Edition

Theme +4: MusicMath

+9 Glossary and Materials List

Glossary

Arithmetic progression A sequence that has a constant difference between terms. For example, 2, 4, 6, 8 has a difference of 2 between the terms.

Beat The regular pulse of music, which may be dictated by the rise or fall of the hand or baton of the conductor.

Combination A way to arrange elements of a group in which the order is not important.

Composition Any musical work. It can also refer to the act of compiling and inventing a piece of music.

Count Musicians keep time by counting beats. Mathematicians count the units of a group or collection in order to determine a total number.

Equation A mathematical expression where the total of everything on the left side of the equal sign (=) has the same value as the total of everything on the right. For example, 6 + 2 = 8 is an equation. The expression C + 2 = 8 is an equation if C= 6.

Fractions A way of expressing a quantity as parts of a whole. If a pie is cut into 4 equal parts, each is expressed as ¼. If it is cut into 8 equal parts, each is ⅛.

Measure In music, beats are grouped into measures (also called bars). In mathematics, measure describes a dimension or quantity.

Model An equation, group of equations, patterns, or data that represent real-world phenomena.

Pattern A pattern is a model, or set of rules, that creates order. A song may have a pattern of sounds or words that are repeated in different stanzas.

Percussion - Instruments that are sounded by striking, shaking, plucking, or scraping. All instruments such as drums and bells fall into this category.

Permutation A way to arrange elements of a group in which the order is important. Phrases - A series of measures.

Repetition An event happens that has happened before. In math, the series of numbers 1, 2, 3, 4, 1, 2, 3, 4 is the sequence 1, 2, 3, 4 repeated twice. In music, a series of sounds or notes may be repeated to create a song.

Rhythm The basic, repetitive pulse of the music or a rhythmic pattern that is repeated throughout the music.

Tempo The speed of the rhythm of a composition. Tempo is measured according to beats per minute. A very fast tempo, called prestissimo, has between 200 and 208 beats per minute, the slowest tempo, called largo, has 40 to 60.

Materials List

- ☐ 8-inch squares of cardboard
- ☐ 8-inch squares of paper in various colors
- ☐ A piece of sheet music
- ☐ A short pointer, or long pencil—to act as a baton
- ☐ Cardboard
- ☐ Chart paper
- ☐ Clothespins or paper clips
- ☐ Construction paper
- ☐ Glue
- ☐ Glue sticks
- ☐ Markers
- ☐ Metronome (optional)
- ☐ Music
- ☐ Paper
- ☐ Pencils
- ☐ Scissors
- ☐ Sticky notes in different colors (optional)
- ☐ String or yarn
- ☐ Tape
- ☐ Wax-covered yarn or pipe cleaners